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Research Article

ESTIMATES OF THE CONCENTRATION FUNCTION FOR STATISTICS $\tilde{T}_n$

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ABSTRACT

The results of this work were published in the journal Reports of the Academy of Sciences of the Republic of Uzbekistan without proof. In this paper, all the results obtained with complete proofs. Several theorems for a linear combination of functions of order statistics are proved in this work.

KEYWORDS

Superpositions of functions, inverse function, Kolmogorov-Smirnov transformation, random variables, estimation of concentration functions, Chebyshev's inequalities, linear combination of order statistics composed of uniform distribution, beta distribution.

INTRODUCTION

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CONSIDER THE STATISTICS $\tilde{T}_n=\sum_{i=1}^{n}c_{in}h(x_{i,n})$. Define the function $\bar{H}(\cdot)$ using the superposition of functions $h(\cdot)$ and $F^{-1}(\cdot)$,. by equality $\bar{H}(\cdot)=h(F^{-1}(\cdot))$. Here $F^{-1}(\cdot)$ - the inverse function to $F(\cdot), F(x)=P(x_1<x)$. Using the Kolmogorov-Smirnov transformation, we can make sure that. $\tilde{T}_n$ и $\sum_{i=1}^{n}c_{in}\bar{H}(u_{in})$ equally distributed[8]. In fact, if $F(x)$ is continuous, then according to the Kolmogorov-Smirnov transformation $F(x_i)$ represents a uniform S.V. u_i on [0,1]. Therefore $F(x_{i,n})=u_{i,n}$. Say, by virtue of continuity $F(x)$, $F^{-1}(u_{i,n})=x_{i,n}$. Therefore $h(x_{i,n})=h(F^{-1}(u_{i,n}))=\bar{H}(u_{i,n})$. Thus $\tilde{T}_n=\sum_{i=1}^{n}c_{in}\bar{H}(u_{i,n})$. Suppose that $\bar{H}$ has a continuous bounded derivative of the second order.

Analysis of literature on the topic (Literature review). As in [9], we use the representation

$$\tilde{T}_n=\sum_{i=1}^{n}c_{in}\bar{H}(x_{i,n})=\sum_{i=1}^{n}c_{in}\bar{H}(u_{i,n})=\mu_n+\bar{U}_n+\bar{R}_n \qquad (1)$$

where

$$\mu_n=\sum_{i=1}^{n}c_{in}\bar{H}\left(\frac{i}{n+1}\right),\quad u_n=\sum_{i=1}^{n}c_{in}\bar{H}'\left(\frac{i}{n+1}\right)\left(u_{i,n}-\frac{i}{n+1}\right),$$

$$\bar{R}_n=\frac{1}{2}\sum_{i=1}^{n}c_{in}\bar{H}''(\theta_{in})\left(u_{i,n}-\frac{i}{n+1}\right)^2,\quad \theta_{in}=\frac{i}{n+1}+\theta\left(u_{i,n}-\frac{i}{n+1}\right),$$

$$|\theta|\le 1,\quad Mu_{i,n}=\frac{i}{n+1}.$$

We investigate the evaluation of the concentration function of S.V. $\tilde{T}_n$, т.е.

$$Q(\tilde{T}_n;\lambda)=\sup_{-\infty<x<\infty}P\{x\le\tilde{T}_n\le x+\lambda\}$$

at any $\lambda\ge 0$. Using (1) and the fact that $Q(\xi+const;\lambda)=Q(\xi;\lambda)$ we have

$$Q(\tilde{T}_n;\lambda)=Q(\mu_n+\bar{U}_n+\bar{R}_n;\lambda)=Q(\mu_n+\bar{U}_n+\bar{R}_n;\lambda) \qquad (2)$$

(Research Methodology). Let 's first make sure of the validity of the following statements[10].

Lemma 1. Let x and y be arbitrary S.V. We put

$$\bar{F}(x)=P(x<x),\ \bar{G}(x)=P(x+y<x).$$

Then $\bar{F}(x-\varepsilon)-P(|y|\ge\varepsilon)\le\bar{G}(x)\le\bar{F}(x+\varepsilon)+P(|y|\ge\varepsilon)$ for any $\varepsilon>0$ и x.

The proof of this lemma is given in [6].

Lemma 2. For any $\lambda\ge 0$ и $\varepsilon>0$ inequality is fair

$$Q(\tilde{T}_n;\lambda) \le Q(\bar{U}_n;\lambda) + 2Q(\bar{U}_n;\varepsilon) + 2P(|\bar{R}_n| \ge \varepsilon).$$

Proof. Applying Lemma 1 to $P(x \le \bar{U}_n + \bar{R}_n \le x+\lambda)$ and by virtue of (2) we find that

$$Q(\tilde{T}_n;\lambda) = \sup_{-\infty<x<\infty} P\{x \le \bar{U}_n + \bar{R}_n \le x+\lambda\} =$$

$$= \sup_{-\infty<x<\infty} \{P(\bar{U}_n + \bar{R}_n \le x+\lambda) - P(\bar{U}_n + \bar{R}_n < x)\} \le$$

$$= \sup_{-\infty<x<\infty} \{P(\bar{U}_n + \bar{R}_n \le x+\lambda) - P(\bar{U}_n + \bar{R}_n < x)\} \le \qquad (3)$$

Since $P(\bar{U}_n < x+\lambda+\varepsilon) - P(\bar{U}_n < x-\varepsilon) =$

$$= P(x-\varepsilon < \bar{U}_n \le x) + P(x < \bar{U}_n \le x+\lambda) +$$

$$+P(x+\lambda < \bar{U}_n \le x+\lambda+\varepsilon) = P(x < \bar{U}_n \le x+\varepsilon) +$$

$$+P(x < \bar{U}_n \le x+\lambda) + P(x \le \bar{U}_n \le x+\varepsilon) =$$

$$= P(x \le \bar{U}_n \le x+\lambda) + 2P(x < \bar{U}_n \le x+\varepsilon),$$

then from the relation (3) we get the inequality

$$Q(\tilde{T}_n;\lambda) \le \sup_{-\infty<x<\infty} \left\{P(x \le \bar{U}_n \le x+\lambda) + 2P(x < \bar{U}_n \le x+\varepsilon) + 2P(|\bar{R}_n| \ge \varepsilon)\right\} \le$$

$$\le \sup_{-\infty<x<\infty} P(x \le \bar{U}_n \le x+\lambda) + 2 \sup_{-\infty<x<\infty} P(x < \bar{U}_n \le x+\varepsilon) + 2P(|\bar{R}_n| \ge \varepsilon) =$$

$$= Q(\bar{U}_n;\lambda) + 2Q(\bar{U}_n;\varepsilon) + 2P(|\bar{R}_n| \ge \varepsilon).$$

Lemma 2 is proved. Let 's put $c'_{in} = c_{in}\bar{H}'\left(\frac{i}{n+1}\right)$. Takes place

Theorem 1. Let $c'_{in} \ge c_{in} > 0 \ (i=1,2,\dots,n; \ n=1,2,\dots)$ and

$\max_x |\bar{H}''(x)| \le C < \infty$. Then

$$Q(\tilde{T}_n;\lambda) \le \frac{\lambda C}{\sqrt{n}} + \frac{CB_n^{\frac{1}{2}}}{n^{3/4}} \qquad (4)$$

where $B_n = \sum_{i=1}^{n} c_{in}$.

Proof. Based on Theorem II.2.1 $Q(\bar{U}_n;\lambda) \le \frac{\lambda C}{\sqrt{n}}$

(5)

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$$Q(\bar{U}_n;\varepsilon)\le\frac{\varepsilon C}{\sqrt{n}} \tag{6}$$

Now let's estimate the probability $P(|\bar{R}_n|\ge\varepsilon)$. By virtue of the definition $\bar{R}_n$ and according to the conditions of Theorem 1, we obtain that

$$P(|\bar{R}_n|\ge\varepsilon)=P\left(\frac{1}{2}\left|\sum_{i=1}^{n}c_{in}\bar{H}''(\theta_{in})\left(u_{i,n}-\frac{i}{n+1}\right)^2\right|\ge\varepsilon\right)\le$$

$$\le P\left(\left|\sum_{i=1}^{n}c_{in}\left(u_{i,n}-\frac{i}{n+1}\right)^2\right|\ge\frac{2\varepsilon}{C}\right)$$

Applying Chebyshev's inequality to the latter, we have

$$P(|\bar{R}_n|\ge\varepsilon)\le\frac{C}{2\varepsilon}M\sum_{i=1}^{n}c_{in}\left(u_{i,n}-\frac{i}{n+1}\right)^2\le$$

$$\le\frac{C}{2\varepsilon}\sum_{i=1}^{n}c_{in}M\left(u_{i,n}-\frac{i}{n+1}\right)^2=\frac{C}{2\varepsilon}\sum_{i=1}^{n}c_{in}Du_{i,n} \tag{7}$$

Since (sm. [7]) $Du_{i,n}=\dfrac{i(n-i+1)}{(n+1)^2(n+2)}$,

then it follows from (7) that

$$P(|\bar{R}_n|\ge\varepsilon)\le\frac{C}{2\varepsilon}\sum_{i=1}^{n}c_{in}\frac{i(n-i+1)}{(n+1)^2(n+2)} \tag{8}$$

Нетрудно убедиться в том, что при $1\le i\le n$

$$\frac{i(n-i+1)}{(n+1)^2(n+2)}\ge\frac{1}{4(n+2)} \tag{9}$$

Consequently, from the relations (8) and (9) we obtain the following inequality

$$P(|\bar{R}_n|\ge\varepsilon)\le\frac{C}{8\varepsilon n}\sum_{i=1}^{n}c_{in}=\frac{C}{8\varepsilon n}B_n \tag{10}$$

In turn, from the relations (5), (6) and (10) according to Lemma 2 we have

$$Q(\tilde{T}_n;\lambda)\le\frac{\lambda C}{\sqrt{n}}+\frac{2\varepsilon C}{\sqrt{n}}+\frac{CB_n}{4\varepsilon n} \tag{11}$$

Minimizing the last two terms on the right side of the relation

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(10) regarding ε, we find, what $\varepsilon = \varepsilon_n = \dfrac{\sqrt{B_n}}{2\sqrt{2}n^{1/4}}$. Putting the found expression for ε and (11) we finally conclude that $\quad Q(\tilde{T}_n;\lambda) \le \dfrac{\lambda C}{\sqrt{n}} + \dfrac{CB_n^{\frac{1}{2}}}{n^{3/4}}$.

Theorem 1 has been fully proved.

Remark 1. Note that the right side of relation (4) tends to zero if $B_n = o(n^{3/2})$.

As the following theorem shows, in the case when the extreme terms of the variation series are removed from the considered linear combination, the conditions on the function $\bar{H}(\cdot)$ may be weakened.

Theorem 2. Let $c_{in} = 0$, if $i < \alpha n$ and $i < \beta n$, a for the rest i, $c_{in} \ge c > 0$. Пусть $\bar{H}$ has continuous derivatives $\bar{H}', \bar{H}''$ в $[F^{-1}(\alpha) - \tau, F^{-1}(\beta) + \tau]$ (τ – arbitrary number). Then $Q(\tilde{T}_n;\lambda) \le \dfrac{\lambda C}{\sqrt{n}} + \dfrac{CB_n^{\frac{1}{2}}}{n^{3/4}}$ где $B_n = \sum_{i=1}^{n} c_{in}$.

Proof. By the condition of the theorem $c_{in} \ne 0$ only in the case when the inequality holds $\alpha < \dfrac{i}{n} < \beta$. Let $\bar{\delta} > 0$ and $\bar{\delta} < \min(\alpha, 1-\beta)$. Let 's put $A = \left\{ \omega : \max_{1 \le i \le n} \left| u_{i,n} - \dfrac{i}{n+1} \right| > \bar{\delta} \right\}$. We show that $P(A) = O\left(\dfrac{1}{n}\right)$..

It is known that (sm. ratio (2.1.1)) $\quad u_{i,n} = \dfrac{z_1 + z_2 + \ldots + z_i}{z_1 + \ldots + z_{n+1}}$, где $z_1, z_2, \ldots, z_{n+1}$ - independent S.V. with general F.R. $G(x) = \max(0, 1 - e^{-x})$. According to the latter

$$A = \left\{ \omega : \max_{1 \le i \le n} \left| u_{i,n} - \frac{i}{n+1} \right| > \bar{\delta} \right\} =$$

$$= \left\{ \omega \max_{1 \le i \le n} \left| \frac{(n+1)(z_1 + z_2 + \ldots + z_i) - i(z_1 + \ldots + z_{n+1})}{(n+1)(z_1 + \ldots + z_{n+1})} \right| > \bar{\delta} \right\} =$$

$$= \left\{ \omega \max_{1 \le i \le n} \left| \frac{(n+1)(z_1 + \ldots + z_i) - i(z_1 + \ldots + z_{n+1})}{(n+1)^2} \right| \frac{n+1}{(z_1 + z_2 + \ldots + z_{n+1})} > \bar{\delta} \right\} \subset$$

$$\subset \left\{ \omega \max_{1 \le i \le n} \left| \frac{(n+1)(z_1 + \ldots + z_i) - i(z_1 + \ldots + z_{n+1})}{(n+1)^2} \right| > \frac{\bar{\delta}}{2} \right\} \cup$$

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$$\cup\left\{\omega:\frac{n+1}{z_1+\ldots+z_{n+1}}>2\right\} \qquad (12)$$

By Chebyshev's inequality, we have

$$P\left(\frac{n+1}{z_1+z_2+\ldots+z_{n+1}}>2\right)\le P\left(\left|\frac{z_1+z_2+\ldots+z_{n+1}}{(n+1)}\right|>\frac{1}{2}\right)=O\left(\frac{1}{n}\right).$$

Therefore

$$P(A)\le P\left(\max_{1\le i\le n}\left|\frac{(n+1)(z_1+\ldots+z_i)-i(z_1+\ldots+z_{n+1})}{(n+1)^2}\right|>\frac{\delta}{2}\right)+O\left(\frac{1}{n}\right) \qquad (13)$$

Let's introduce an event into consideration

$$A_i=\left\{\omega:\left|\frac{(n+1)(z_1+\ldots+z_i)-i(z_1+\ldots+z_{n+1})}{(n+1)^2}\right|>\frac{\delta}{2}\right\}.$$

Since the event $\left\{\frac{|(n+1)(z_1+\ldots+z_i)-i(z_1+\ldots+z_{n+1}-(n+1))|}{(n+1)^2}>\frac{\delta}{2}\right\}\subset$

$$\subset\left\{\frac{|(n+1)(z_1+\ldots+z_i-i)|}{(n+1)^2}>\frac{\delta}{2}\right\}\cup\left\{\frac{|i(z_1+\ldots+z_i-(n+1))|}{(n+1)^2}>\frac{\delta}{4}\right\}.$$

Then, it follows that

$$P(A_i)=P\left\{\frac{|(n+1)(z_1+\ldots+z_i-i)-i(z_1+\ldots+z_{n+1}-(n+1))|}{(n+1)^2}>\frac{\delta}{2}\right\}\le$$

$$\le P\left(\frac{|z_1+\ldots+z_i-i|}{(n+1)}>\frac{\delta}{4}\right)+P\left(\frac{i\,|z_1+\ldots+z_{n+1}-(n+1)|}{(n+1)^2}>\frac{\delta}{4}\right) \qquad (14)$$

$$\le P\left(\frac{|z_1+\ldots+z_i-i|}{(n+1)}>\frac{\delta}{4}\right)+P\left(\frac{|z_1+\ldots+z_{n+1}-(n+1)|}{(n+1)^2}>\frac{\delta}{4}\right)$$

Further, again by virtue of Chebyshev's inequality, we have

$$P(A_i)\le P\left(\frac{|z_1+\ldots+z_i-M(z_1+\ldots+z_i)|}{(n+1)}>\frac{\delta}{4}\right)+$$

$$+P\left(\frac{|z_1+\ldots+z_{n+1}-M(z_1+\ldots+z_{n+1})|}{(n+1)}>\frac{\delta}{4}\right)\le$$

$$\le \frac{C}{(n+1)^4} M\left[\sum_{j=1}^{i}(z_i - Mz_i)\right]^4 + \frac{C}{(n+1)^4} M\left[\sum_{j=1}^{i}(z_i - Mz_i)\right]^4 \qquad (15)$$

Applying inequalities for moments of sums of independent S.V. given in V.V.Petrov's monograph ([5] page. 79) from (15) we get that $P(A_i) \le \frac{C}{(n+1)^2}$.

By virtue of the latter, from the relation (13) we find that

$$P(A) \le P\left(\bigcup_{i=1}^{n} A_i\right) + O\left(\frac{1}{n}\right) \le \sum_{i=1}^{n} P(A_i) + O\left(\frac{1}{n}\right) = O\left(\frac{1}{n}\right).$$

Say, we have shown that $P(A) = O\left(\frac{1}{n}\right)$. Note that if an event occurs $\bar{A}$, that is , the values

$\theta_{in} = \frac{i}{n+1}\theta\left(u_{i,n} - \frac{i}{n+1}\right)$, included in expression (1) satisfy the obvious inequality

$$0 < \alpha - \bar{\delta} \le \theta_{in} \le \beta + \bar{\delta} < 1.$$

From this, according to the conditions of Theorem 2, if an event occurs $\bar{A}$, it follows that $|H''(\theta_{in})| \le C$. Further reasoning coincides with the reasoning of Theorem 1 Theorem 2 is proved.

Remark 2. Suppose $h(x) = x$. Then

$$H(x) = F^{-1}(x),\ H'(x) = \frac{1}{p(F^{-1}(x))},\ H''(x) = -\frac{p'(F^{-1}(x))}{p^3(F^{-1}(x))},$$

где $p(x) = \frac{d}{dx}F(x)$. If $p(x) = F'(x)$ exists $p(x) \ne 0$, with $p(x)$, $p'(x)$ - continuous and $|p'(x)| < C$, then the conditions of Theorem 1 are fulfilled[2, 3].

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