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Analysis Of Stress-Strain State Longitudinal-Radial Vibrations Of Transversely-Isotropic Cylindrical Shells Interacting Non-Stationary With An Internal Viscous Fluid

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Abstract: In this work, the longitudinal-radial vibrations of a transversely-isotropic cylindrical shell interacting with an internal viscous fluid are mathematically modeled. The interactions between the shell and the fluid are considered on the basis of refined equations of motion. Fourier and Laplace transformations, boundary and contact conditions, and the finite difference method are used to solve the equations. Displacement graphs are constructed based on a practical problem and their correspondence to the physical essence is shown. The results of the research are of great importance in the design and optimization of industrial structures.

Keywords: Transversally-isotropic cylindrical shell, viscous fluid, longitudinal-radial vibrations, displacement analysis, boundary conditions, contact conditions.

INTRODUCTION:

Elastic vibrations of cylindrical shells and their interaction with internal fluids are important in oil and gas pipelines, jet engines, and underwater structures. Often, the interaction with fluids in these processes is not studied. Therefore, improved mathematical models are needed to determine the longitudinal-radial vibrations of transversally-isotropic cylindrical shells.

Scientific research conducted in the field of modeling longitudinal-radial vibrations of transversely-isotropic cylindrical shells in the non-stationary interaction with an internal viscous fluid forms the theoretical basis of this work.

In recent years, much attention has been paid to the modeling of longitudinal-radial vibrations arising from the interaction of transversely isotropic cylindrical shells with an internal viscous fluid in engineering. Initial models describing the propagation of longitudinal and transverse waves in shells have been developed [1]. Based on them, the propagation of sound waves inside cylindrical shells has also been theoretically studied [2].

Accurate three-dimensional mathematical computational models, taking into account the interaction of the fluid and the shell, are being created on the basis of classical and improved approximate vibration equations [3-5]. In this work, based on the refined equations, a system of longitudinal-radial vibration equations of a cylindrical shell interacting with an internal viscous fluid is formed. By solving the resulting system of equations, the deformed state of an arbitrary point of the cylindrical shell interacting with an internal viscous fluid is determined [6-11].

METHODS

The length l of the transversely-isotropic cylindrical shell under study, let the outer radius be r_2 and the inner radius be r_1. We consider the problem of the unsteady interaction of this cylindrical shell with an internal viscous fluid.

The components of the stress tensor for a transversely isotropic body in a cylindrical coordinate system are as follows:

$$\sigma_{rr}=C_{11}\frac{\partial U_r}{\partial r}+C_{12}\frac{U_r}{r}+C_{13}\frac{\partial U_z}{\partial z};\sigma_{\theta\theta}=C_{12}\frac{\partial U_r}{\partial r}+C_{11}\frac{U_r}{r}+C_{13}\frac{\partial U_z}{\partial z};\sigma_{zz}=C_{13}\left(\frac{\partial U_r}{\partial r}+\frac{U_r}{r}\right)+C_{33}\frac{\partial U_z}{\partial z};\quad (1)$$

where C_{ij} are elastic invariants for a transversely isotropic body, U_r and U_z are radial components of displacement vectors.

The equations of motion for longitudinal-radial vibration in a cylindrical coordinate system are written in the following form:

$$\begin{aligned}&C_{11}\frac{\partial^2 U_r}{\partial r^2}-\frac{C_{12}}{r^2}U_r+\frac{C_{12}}{r}\frac{\partial U_r}{\partial r}+C_{13}\frac{\partial^2 U_z}{\partial r\partial z}+C_{44}\left(\frac{\partial^2 U_r}{\partial^2 z}+\frac{\partial^2 U_z}{\partial r\partial z}\right)+\frac{C_{11}-C_{12}}{r}\frac{\partial U_r}{\partial r}+\frac{C_{12}-C_{11}}{r^2}U_r=\rho\frac{\partial^2 U_r}{\partial t^2};\\&C_{44}\left(\frac{\partial^2 U_r}{\partial r\partial z}+\frac{\partial^2 U_z}{\partial r^2}\right)+C_{13}\left(\frac{\partial^2 U_r}{\partial z\partial r}+\frac{1}{r}\frac{\partial U_r}{\partial r}\right)+C_{33}\frac{\partial^2 U_z}{\partial z^2}+C_{44}\left(\frac{1}{r}\frac{\partial U_r}{\partial z}+\frac{1}{r}\frac{\partial U_z}{\partial r}\right)=\rho\frac{\partial^2 U_z}{\partial t^2};\end{aligned}\quad (2)$$

Applying the Fourier-Laplace substitution to the system of equations (2), we obtain the following system of equations

$$\begin{aligned}&\tilde{C}_{11}\left(\frac{\partial^2\tilde{U}_r}{\partial r^2}+\frac{1}{r}\frac{\partial\tilde{U}_r}{\partial r}-\frac{1}{r^2}\tilde{U}_r\right)+\tilde{C}_{44}k^2\tilde{U}-k\left(\tilde{C}_{13}+\tilde{C}_{44}\right)\frac{\partial^2\tilde{U}_z}{\partial r}=\rho p^2\tilde{U}_r;\\&\tilde{C}_{44}\left(\frac{\partial^2\tilde{U}_z}{\partial r^2}+\frac{1}{r}\frac{\partial\tilde{U}_z}{\partial r}\right)-k^2\tilde{C}_{33}\tilde{U}_z-k\left(\tilde{C}_{44}+\tilde{C}_{13}\right)\left(\frac{\partial\tilde{U}_r}{\partial r}+\frac{\tilde{U}}{r}\right)=\rho p^2\tilde{U}_z;\end{aligned}\quad (3)$$

Let us introduce the following symbols

$$\tilde{\Delta}_0=\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\frac{1}{r^2};\quad \tilde{B}_1=\frac{\tilde{C}_{13}+\tilde{C}_{44}}{\tilde{C}_{11}};\quad \tilde{B}_2=\frac{\tilde{C}_{13}+\tilde{C}_{44}}{\tilde{C}_{44}};\quad (4)$$

given definitions (4), the system of equations (3) can be written, differentiated with respect to r, in the following form:

$$\tilde{\Delta}_0\tilde{U}_r+\left(\frac{\tilde{C}_{44}}{\tilde{C}_{11}}k^2-\frac{\rho}{\tilde{C}_{11}}p^2\right)\tilde{U}_r-k\tilde{B}_1\frac{\partial\tilde{U}_z}{\partial r}=0;\ \tilde{\Delta}_0\frac{\partial\tilde{U}_z}{\partial r}-\left(\frac{\tilde{C}_{33}}{\tilde{C}_{44}}k^2+\frac{\rho}{\tilde{C}_{44}}p^2\right)\frac{\partial\tilde{U}_z}{\partial r}-k\tilde{B}_2\tilde{\Delta}_0\tilde{U}_r=0;\quad (5)$$

From the system of equations (5) get the following:

$$\left(\tilde{\Delta}-\alpha_1^2\right)\left(\tilde{\Delta}-\alpha_2^2\right)\tilde{U}_r=0,\quad (6)$$

here

$$\alpha_1^2+\alpha_2^2=\rho\left(\frac{1}{\tilde{C}_{11}}+\frac{1}{\tilde{C}_{44}}\right)p^2+\left(-\frac{\tilde{C}_{44}}{\tilde{C}_{11}}+\frac{\tilde{C}_{33}}{\tilde{C}_{44}}+\frac{\left(\tilde{C}_{13}+\tilde{C}_{44}\right)^2}{\tilde{C}_{11}\tilde{C}_{44}}\right)k^2;\alpha_1^2\alpha_2^2=-k^4\frac{\tilde{C}_{33}}{\tilde{C}_{11}}+\frac{\rho^2p^4}{\tilde{C}_{11}\tilde{C}_{44}}+\frac{\rho p^2k^2}{\tilde{C}_{11}}\left(\frac{\tilde{C}_{33}}{\tilde{C}_{44}}-1\right);\quad (7)$$

The general solution of equation (6) is equal to the sum of the solutions of the following equation [11]

$$\frac{d^2U_i}{dr^2}+\frac{1}{r}\frac{dU_i}{dr}-\left(\alpha_i^2+\frac{1}{r^2}\right)U_i=0\qquad (i=1,2),\quad (8)$$

The general solution of equation (8) is equal to

$$\tilde{U}_r=A_1I_1(\alpha_1 r)+D_1K_1(\alpha_1 r)+A_2I_1(\alpha_2 r)+D_2K_1(\alpha_2 r)\quad (9)$$

Substituting solution (9) into the first equation of system (5), we integrate with respect to r and obtain the following.

$$k\tilde{B}_1\tilde{U}_z=\frac{\alpha_1^2-\alpha^2}{\alpha_1}\left[A_1I_0(\alpha_1 r)+D_1K_0(\alpha_1 r)\right]+\frac{\alpha_2^2-\alpha^2}{\alpha_1}\left[A_2I_0(\alpha_2 r)+D_2K_0(\alpha_2 r)\right]\quad (10)$$

Expanding the Bessel functions in expressions (9) and (10) into power series with respect to the radial coordinate r, we obtain the following.

$$\begin{aligned}&\tilde{U}_z=\sum_{n=0}^{\infty}\left(\frac{\alpha_1^2-\alpha^2}{k\alpha_1\tilde{B}_1}\alpha_1^{2n}A_{10}+\frac{\alpha_2^2-\alpha^2}{k\alpha_2\tilde{B}_1}\alpha_2^{2n}A_{20}\right)\frac{(r/2)^{2n}}{(n!)^2}+\sum_{n=0}^{\infty}\eta_{6,n}(r)\left(\frac{\alpha_1^2-\alpha^2}{k\alpha_1\tilde{B}_1}\alpha_1^{2n}D_1+\frac{\alpha_2^2-\alpha^2}{k\alpha_2\tilde{B}_1}\alpha_2^{2n}D_2\right)\frac{(r/2)^{2n}}{(n!)^2};\\&\tilde{U}_r=\frac{1}{r}\left(\frac{D_1}{\alpha_1}+\frac{D_2}{\alpha_2}\right)+\sum_{n=0}^{\infty}\left(A_{10}\alpha_1^{2n+1}+A_{20}\alpha_2^{2n+1}\right)\frac{(r/2)^{2n+1}}{n!(n+1)!}+\sum_{n=0}^{\infty}\eta_{7,n}(r)\left(D_1\alpha_1^{2n+1}+D_2\alpha_2^{2n+1}\right)\frac{(r/2)^{2n+1}}{n!(n+1)!};\end{aligned}\quad (11)$$

here

$A_{10}=A_1+D_1\left[\ln\frac{\alpha_1\xi}{2}-\psi(1)\right];A_{20}=A_2+D_2\left[\ln\frac{\alpha_2\xi}{2}-\psi(1)\right];\eta_{6,n}(r)=\ln\frac{r}{\xi}-\sum_{k=1}^{n}\frac{1}{k};\eta_{7,n}(r)=\ln\frac{r}{\xi}-\sum_{k=1}^{n}\frac{1}{k}-\frac{1}{2(n+1)}.$ For longitudinal-radial vibrations of a transversely isotropic cylindrical shell, the following boundary conditions and kinematic conditions apply:

$$\sigma_{rr}(r,z,t)\big|_{r=r_2}=f_r(z,t)\big|_{r=r_2},\sigma_{rz}(r,z,t)\big|_{r=r_2}=f_r(z,t)\big|_{r=r_2},\sigma_{rr}(r,z,t)\big|_{r=r_1}=-P_{rr}(r,z,t)\big|_{r=r_1},\sigma_{rz}(r,z,t)\big|_{r=r_1}=-P_{rz}(r,z,t)\big|_{r=r_1}. \quad (12)$$

(12) Using Fourier-Laplace transforms for both boundary and contact conditions $\partial U_r/\partial t=\upsilon_r,\partial U_z/\partial t=\upsilon_z$, we obtain the following:

$$\begin{aligned}
&\tilde{C}_{11}\frac{\partial\tilde{U}_r}{\partial r}+\tilde{C}_{12}\frac{\tilde{U}_r}{r}-k\tilde{C}_{13}\tilde{U}_z=f_r(k,t);\\
&\tilde{C}_{44}\left(k\tilde{U}_r+\frac{\partial\tilde{U}_z}{\partial r}\right)=f_{rz}(k,t);\\
&\left(\tilde{C}_{11}+\frac{1}{3}\tilde{\mu}_s p\right)\frac{\partial\tilde{U}_r}{\partial r}+\left(\tilde{C}_{12}-\frac{2}{3}\tilde{\mu}_s p\right)\frac{\tilde{U}_r}{r}-\left(k\tilde{C}_{13}+\frac{2}{3}\tilde{\mu}_s pk\right)\tilde{U}_z=\tilde{p}_s;\\
&\left(\tilde{C}_{44}+\frac{1}{2}\tilde{\mu}_s p\right)\left(k\tilde{U}_r+\frac{\partial\tilde{U}_z}{\partial r}\right)=0;
\end{aligned} \quad (13)$$

Let us substitute expressions (9) and (10) into this system of equations (13) and replace the principal parts of their displacements and coefficients α^2, α_1^2 and α_2^2 (alpha) as follows:

$$\begin{aligned}
&\left[U_{r,0};U_{r,1};\lambda^2;\lambda_2^n;\lambda_1^{2n}\right]=\int_0^\infty\left.\begin{matrix}\sin kz\\-\cos kz\end{matrix}\right\}dk\int_{(l)}\left[\tilde{U}_{r,0};\tilde{U}_{r,1};\lambda_1^{2n};(\alpha_1^{2n}+\alpha_2^{2n});\alpha_1^{2n}\alpha_2^{2n}\right]e^{pt}dp;\\
&\left[U_{z,0};U_{z,1}\right]=\int_0^\infty\left.\begin{matrix}\cos kz\\\sin kz\end{matrix}\right\}dk\int_{(l)}\left[\tilde{U}_{z,0};\tilde{U}_{z,1}\right]e^{pt}dp;
\end{aligned} \quad (14)$$

(14) in the system of equations obtained as a result of applying the substitutions

$$\begin{aligned}
&\lambda=C_{11}^{-1}\left(\rho\frac{\partial^2}{\partial t^2}-C_{44}\frac{\partial^2}{\partial z^2}\right);\\
&\lambda_1^2=C_{11}^{-1}\left(\rho^2C_{44}^{-1}\frac{\partial^4}{\partial t^4}-\rho\left(1+C_{33}C_{44}^{-1}\right)\frac{\partial^4}{\partial t^2\partial z^2}+C_{33}\frac{\partial^4}{\partial z^4}\right);\\
&\lambda_2=\rho\left(C_{11}^{-1}+C_{44}^{-1}\right)\frac{\partial^2}{\partial t^2}-\left(C_{44}C_{11}^{-1}+C_{44}^{-1}C_{33}+C_{44}^{-1}C_{11}^{-1}\left(C_{13}+C_{44}\right)^2\right)\frac{\partial^2}{\partial z^2};
\end{aligned}$$

Substituting the values of the operators and performing mathematical simplifications, we obtain the following system of equations:

$$\begin{aligned}
&\left(a_{11}\frac{\partial^4}{\partial t^4}+a_{12}\frac{\partial^4}{\partial t^2\partial z^2}+a_{13}\frac{\partial^4}{\partial z^4}+a_{14}\frac{\partial^2}{\partial t^2}+a_{15}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,0}+\left(b_{11}\frac{\partial^4}{\partial t^4}+b_{12}\frac{\partial^4}{\partial t^2\partial z^2}+b_{13}\frac{\partial^4}{\partial z^4}+b_{14}\frac{\partial^2}{\partial t^2}+\right.\\
&\left.+b_{15}\frac{\partial^2}{\partial z^2}\right)U_{r,0}+\left(n_{11}\frac{\partial^4}{\partial t^4}+n_{12}\frac{\partial^4}{\partial t^2\partial z^2}+n_{13}\frac{\partial^4}{\partial z^4}+n_{14}\frac{\partial^2}{\partial t^2}+n_{15}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,1}+\left(m_{11}\frac{\partial^4}{\partial t^4}+m_{12}\frac{\partial^4}{\partial t^2\partial z^2}+\right.\\
&\left.+m_{13}\frac{\partial^4}{\partial z^4}+m_{14}\frac{\partial^2}{\partial t^2}+m_{15}\frac{\partial^2}{\partial z^2}\right)U_{r,1}=\left(s_{11}\frac{\partial^2}{\partial t^2}+s_{12}\frac{\partial^2}{\partial z^2}\right)f_r(z,t);\\
&\left(a_{21}\frac{\partial^4}{\partial t^4}+a_{22}\frac{\partial^4}{\partial t^2\partial z^2}+a_{23}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,0}+\left(b_{21}\frac{\partial^4}{\partial t^2\partial z^2}+b_{22}\frac{\partial^4}{\partial z^4}\right)U_{r,0}+\left(n_{21}\frac{\partial^4}{\partial t^4}+n_{22}\frac{\partial^4}{\partial t^2\partial z^2}+\right.\\
&\left.+n_{23}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,1}+\left(m_{21}\frac{\partial^4}{\partial t^4}+m_{22}\frac{\partial^4}{\partial t^2\partial z^2}+m_{23}\frac{\partial^4}{\partial z^4}\right)U_{r,1}=\left(s_{21}\frac{\partial^3}{\partial z\partial t^2}+s_{22}\frac{\partial^3}{\partial z^3}\right)f_{rz}(z,t); \quad (15)\\
&\left(a_{31}\frac{\partial^4}{\partial t^4}+a_{32}\frac{\partial^4}{\partial t^2\partial z^2}+a_{33}\frac{\partial^4}{\partial z^4}+a_{34}\frac{\partial^3}{\partial t^3}+a_{35}\frac{\partial^3}{\partial t\partial z^2}+a_{36}\frac{\partial^2}{\partial t^2}+a_{37}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,0}+\left(b_{31}\frac{\partial^4}{\partial t^4}+b_{32}\frac{\partial^4}{\partial t^2\partial z^2}+\right.\\
&\left.+b_{33}\frac{\partial^4}{\partial z^4}+b_{34}\frac{\partial^2}{\partial t^2}+b_{35}\frac{\partial^2}{\partial z^2}\right)U_{r,0}+\left(n_{31}\frac{\partial^4}{\partial t^4}+n_{32}\frac{\partial^4}{\partial t^2\partial z^2}+n_{33}\frac{\partial^4}{\partial z^4}+n_{34}\frac{\partial^3}{\partial t^3}+n_{35}\frac{\partial^3}{\partial t\partial z^2}+n_{36}\frac{\partial^2}{\partial t^2}+n_{37}\frac{\partial^2}{\partial z^2}\right)\times
\end{aligned}$$

$$\times\frac{\partial}{\partial z}U_{z,1}+\left(m_{31}\frac{\partial^4}{\partial t^4}+m_{32}\frac{\partial^4}{\partial t^2\partial z^2}m_{33}\frac{\partial^4}{\partial z^4}+m_{34}\frac{\partial^2}{\partial t^2}+m_{35}\frac{\partial^2}{\partial z^2}\right)U_{r,1}=\left(s_{31}\frac{\partial^2}{\partial t^2}+s_{32}\frac{\partial^2}{\partial z^2}\right)p_s;$$

$$\left(a_{41}\frac{\partial^4}{\partial t^4}+a_{42}\frac{\partial^4}{\partial t^2\partial z^2}+a_{43}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,0}+\left(b_{41}\frac{\partial^4}{\partial t^2\partial z^2}+b_{42}\frac{\partial^4}{\partial z^4}\right)U_{r,0}+\left(n_{41}\frac{\partial^4}{\partial t^4}+n_{42}\frac{\partial^4}{\partial t^2\partial z^2}+\right.$$

$$\left.+n_{43}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,1}+\left(m_{41}\frac{\partial^4}{\partial t^4}+m_{42}\frac{\partial^4}{\partial t^2\partial z^2}+m_{43}\frac{\partial^4}{\partial z^4}\right)U_{r,1}=0;$$

The coefficients in the system of equations (15) are determined by the geometric and physical parameters of the shell material. For example,

$$a_{11}=\frac{\rho^2B_1r_2^2}{4C_{11}C_{44}}\left(B_1C_{11}-C_{13}+\frac{C_{12}}{4}\right),...,b_{11}=\left(\frac{\rho^2r_2^2}{4C_{11}C_{44}}\left(B_1C_{11}+C_{13}-\frac{B_1C_{12}}{4}\right)+\frac{\rho^2}{C_{11}}\left(\frac{1}{C_{11}}+\frac{1}{C_{44}}\right)\left(B_1C_{11}+\frac{C_{12}}{4}B_1-C_{13}\right)\frac{r_2^2}{4}+\frac{\rho^2r_2^2C_{13}}{4C_{11}^2}\right),...,$$

$$n_{11}=\xi\left(\frac{\rho^2B_1}{C_{11}}\left(\frac{1}{C_{11}}+\frac{1}{C_{44}}\right)\left(\eta_{2,1}(r_2)\left(B_1C_{11}-C_{13}\right)+\eta_{1,1}(r_2)\frac{B_1C_{12}}{4}\right)\frac{r_2^2}{4}+\eta_{2,1}(r_2)\frac{\rho^2r_2^2C_{13}B_1}{4C_{11}^2}\right),...,$$

$$m_{11}=\frac{B_1\xi\rho^2}{C_{11}^2}\left(\eta_{2,0}(r_2)C_{11}+\eta_{1,0}(r_2)\frac{C_{12}}{2}\right),...,\ s_{11}=\frac{\rho B_1}{C_{11}},....$$

By solving this system of equations (15), it is possible to find functions $U_{r,0}$, $U_{z,0}$, $U_{r,1}$ and $U_{z,1}$ that allow determining the displacements occurring at the points of their cross sections during unsteady vibrations of circular cylindrical layers and shells interacting with an internal viscous fluid.

RESULTS

Let us now, based on the system of equations (15), we consider a practical problem in the longitudinal radial vibrations of circular cylindrical transversely-isotropic layers and shells interacting with an internal viscous fluid. We introduce the following dimensionless quantities

$$r=lr^*;\ z=lz^*;\ C_{11}=CC_{11}^*;\ t=\sqrt{\frac{\rho}{C}}lt^*;U_{z,0}=lU_{z,0}^*;U_{r,0}=U_{r,0}^*;U_{z,1}=U_{z,1}^*;U_{r,1}=lU_{r,1}^*; \quad (16)$$

By introducing dimensionless definitions (16) into the system of equations (15), we obtain a system of equations in the dimensionless state

$$\left(\bar a_{11}\frac{\partial^4}{\partial t^4}+\bar a_{12}\frac{\partial^4}{\partial t^2\partial z^2}+\bar a_{13}\frac{\partial^4}{\partial z^4}+\bar a_{14}\frac{\partial^2}{\partial t^2}+\bar a_{15}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,0}+\left(\bar b_{11}\frac{\partial^4}{\partial t^4}+\bar b_{12}\frac{\partial^4}{\partial t^2\partial z^2}+\bar b_{13}\frac{\partial^4}{\partial z^4}+\right.$$

$$\left.+\bar b_{14}\frac{\partial^2}{\partial t^2}+\bar b_{15}\frac{\partial^2}{\partial z^2}\right)U_{r,0}+\left(\bar n_{11}\frac{\partial^4}{\partial t^4}+\bar n_{12}\frac{\partial^4}{\partial t^2\partial z^2}+\bar n_{13}\frac{\partial^4}{\partial z^4}+\bar n_{14}\frac{\partial^2}{\partial t^2}+\bar n_{15}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,1}+$$

$$+\left(\bar m_{11}\frac{\partial^4}{\partial t^4}+\bar m_{12}\frac{\partial^4}{\partial t^2\partial z^2}+\bar m_{13}\frac{\partial^4}{\partial z^4}+\bar m_{14}\frac{\partial^2}{\partial t^2}+\bar m_{15}\frac{\partial^2}{\partial z^2}\right)U_{r,1}=\left(\bar s_{11}\frac{\partial^2}{\partial t^2}+\bar s_{12}\frac{\partial^2}{\partial z^2}\right)f_r;$$

$$\left(\bar a_{21}\frac{\partial^4}{\partial t^4}+\bar a_{22}\frac{\partial^4}{\partial t^2\partial z^2}+\bar a_{23}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,0}+\left(\bar b_{21}\frac{\partial^4}{\partial t^2\partial z^2}+\bar b_{22}\frac{\partial^4}{\partial z^4}\right)U_{r,0}+\left(\bar n_{21}\frac{\partial^4}{\partial t^4}+\bar n_{22}\frac{\partial^4}{\partial t^2\partial z^2}+\right.$$

$$\left.+\bar n_{23}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,1}+\left(\bar m_{21}\frac{\partial^4}{\partial t^4}+\bar m_{22}\frac{\partial^4}{\partial t^2\partial z^2}+\bar m_{23}\frac{\partial^4}{\partial z^4}\right)U_{r,1}=\left(\bar s_{21}\frac{\partial^3}{\partial z\partial t^2}+\bar s_{22}\frac{\partial^3}{\partial z^3}\right)f_{rz}; \quad (17)$$

$$\left(\bar a_{31}\frac{\partial^4}{\partial t^4}+\bar a_{32}\frac{\partial^4}{\partial t^2\partial z^2}+\bar a_{33}\frac{\partial^4}{\partial z^4}+\bar a_{34}\frac{\partial^3}{\partial t^3}+\bar a_{35}\frac{\partial^3}{\partial t\partial z^2}+\bar a_{36}\frac{\partial^2}{\partial t^2}+\bar a_{37}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,0}+\left(\bar b_{31}\frac{\partial^4}{\partial t^4}+\bar b_{32}\frac{\partial^4}{\partial t^2\partial z^2}+\right.$$

$$\left.+\bar b_{33}\frac{\partial^4}{\partial z^4}+\bar b_{34}\frac{\partial^2}{\partial t^2}+\bar b_{35}\frac{\partial^2}{\partial z^2}\right)U_{r,0}+\left(\bar n_{31}\frac{\partial^4}{\partial t^4}+\bar n_{32}\frac{\partial^4}{\partial t^2\partial z^2}+\bar n_{33}\frac{\partial^4}{\partial z^4}+\bar n_{34}\frac{\partial^3}{\partial t^3}+\bar n_{35}\frac{\partial^3}{\partial t\partial z^2}+\bar n_{36}\frac{\partial^2}{\partial t^2}+\right.$$

$$\left.+\bar n_{37}\frac{\partial^2}{\partial z^2}\right)\frac{\partial}{\partial z}U_{z,1}+\left(\bar m_{31}\frac{\partial^4}{\partial t^4}+\bar m_{32}\frac{\partial^4}{\partial t^2\partial z^2}+\bar m_{33}\frac{\partial^4}{\partial z^4}+\bar m_{34}\frac{\partial^2}{\partial t^2}+\bar m_{35}\frac{\partial^2}{\partial z^2}\right)U_{r,1}=\left(\bar s_{31}\frac{\partial^2}{\partial t^2}+\bar s_{32}\frac{\partial^2}{\partial z^2}\right)p_s;$$

$$\left(\bar a_{41}\frac{\partial^4}{\partial t^4}+\bar a_{42}\frac{\partial^4}{\partial t^2\partial z^2}+\bar a_{43}\frac{\partial^4}{\partial z^4}\right)\frac{\partial}{\partial z}U_{z,0}+\left(\bar b_{41}\frac{\partial^4}{\partial t^2\partial z^2}+\bar b_{42}\frac{\partial^4}{\partial z^4}\right)U_{r,0}+\left(\bar n_{41}\frac{\partial^4}{\partial t^4}+\bar n_{42}\frac{\partial^4}{\partial t^2\partial z^2}+\right.$$

$$+\bar{n}_{43}\frac{\partial^4}{\partial z^4}\bigg)\frac{\partial}{\partial z}U_{z,1}+\bigg(\bar{m}_{41}\frac{\partial^4}{\partial t^4}+\bar{m}_{42}\frac{\partial^4}{\partial t^2\partial z^2}+\bar{m}_{43}\frac{\partial^4}{\partial z^4}\bigg)U_{r,1}=0;$$

The coefficients of the system of equations (17) represent the dimensionless form of the coefficients of the system of equations (15).

We use the finite difference method to solve the system of equations (17). The geometric dimensions of the circular cylindrical shell to solve the system of equations expanded using the finite difference method using the Maple program are as follows: $l=1m,\ r_1=0.2\ m,\ r_1=0.128\ m.$ Apply torques to the free end of a circular cylindrical shell at point $z=0$. The end $z=l$ is fixed with a single screwWe consider the circular cylindrical shell material as aluminum, zinc, and graphitepoxy. For aluminum material $\rho=2700\ kg/m^3,\ E=0.7\cdot10^{11}\ N/m^2,\ \nu=0.33,\ C_{11}=1.103\cdot10^{11}\ N/m^2,$ $C_{12}=0.543\cdot10^{11}\ N/m^2,\ C_{13}=0.543\cdot10^{11}\ N/m^2,\ C_{33}=1.103\cdot10^{11}\ N/m^2,\ C_{44}=0.280\cdot10^{11}\ N/m^2$, for zinc material $\rho=7140\ kg/m^3,\ C_{11}=1.583\cdot10^{11}\ N/m^2,\ C_{12}=0.315\cdot10^{11}\ N/m^2,\ C_{13}=0.474\cdot10^{11}\ N/m^2,\ C_{33}=0.616\cdot10^{11}\ N/m^2,$ $C_{44}=0.40\cdot10^{11}\ N/m^2$, for graphitepoxy material $\rho=1700\ kg/m^3,\ C_{11}=0.139\cdot10^{11}\ N/m^2,\ C_{12}=0.064\cdot10^{11}\ N/m^2,$ $C_{13}=0.064\cdot10^{11}\ N/m^2,\ C_{33}=1.160\cdot10^{11}\ N/m^2,\ C_{44}=0.070\cdot10^{11}\ N/m^2$ we obtain values. The thickness of the cylindrical shell is $h=r_2-r_1$. The calculations resulted in Figures 1.2.

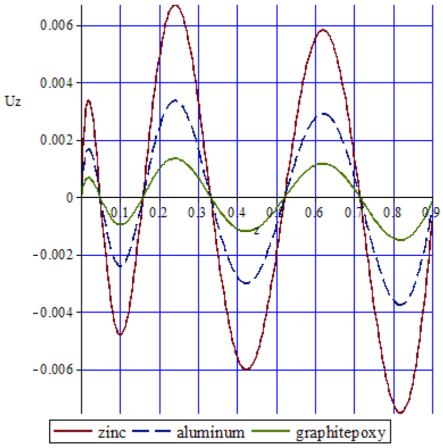

Figure 1. Graph of the change in the displacement vector U_z component as a function of the *z* coordinate when the cylindrical shell material changes.

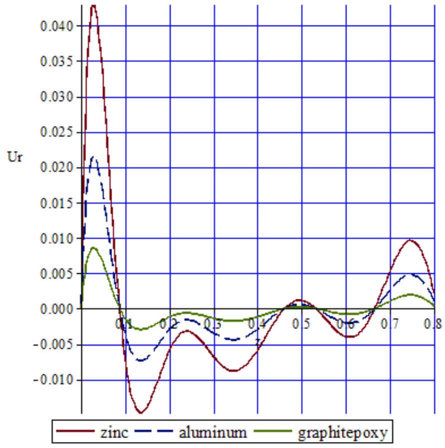

Figure 2. Graph of the change in the displacement vector U_r component as a function of the *z* coordinate when the cylindrical shell material changes.

DISCUSSION

Fig. 1 shows a graph of the change in the displacement vector component U_z as a function of coordinate for zinc, aluminium, and graphitepoxy resin shell materials at a torque $10\cdot10^3 Nm$ applied to the end face of a circular cylindrical shell interacting with an internal viscous fluid. Fig. 2 shows the change in the displacement vector component U_r depending on the coordinates when a torque $10\cdot10^3 Nm$ is applied to the end face of a circular cylindrical shell interacting with an internal viscous fluid, where the shell material is zinc, aluminium, and graphite epoxy resin. Here we also see that as the torque value U_r increases, the displacement value also increases. This corresponds to the physical essence of the problem.

CONCLUSION

This work models longitudinal-radial vibrations of transversely isotropic cylindrical shells filled with an internal viscous fluid and determines the displacements of shell points. Contrary to traditional hypotheses, differential equations for longitudinal and radial displacements in cylindrical coordinates are derived based on refined equations of elastic motion. Taking into account the boundary and kinematic conditions between the shell and the fluid, general solutions are found using Fourier and Laplace transforms. According to the results, with an increase in torque, the displacements reach their maximum values, and these values decay along the length of the shell. The graphical results correspond to the physical essence of the problem and confirm the reliability of the model. The results obtained provide a basis for in-depth study of the dynamic state of cylindrical shells under the influence of an internal viscous fluid and for optimization of such structures.

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