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Calculation and Evaluation of Tourism Indicators Using Mathematical Models

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Abstract: Tourism plays an important role in the economy of many countries, developing infrastructure, creating new jobs and contributing to budget revenues. In this article, various mathematical models, including the Lotka-Volterra model, the Cobb-Douglas production function, the multiplier effect model, the Markov chain model, and dynamic game models, are adapted for tourism resources. For each model, estimated values are provided, and tourism indicators are calculated.

Keywords: Mathematical model of tourism, tourist flow, Lotka-Volterra, Markov chain, Cobb-Douglas function, multiplier effect, dynamic games.

Introduction:

In the Republic of Uzbekistan, various mathematical models can be employed to analyze the sustainable development of tourism. These models integrate the economic, environmental, and social dimensions of the tourism. Mathematical modeling serves as an effective tool for analyzing, forecasting, and managing tourism.

Famous scholars who have studied economic and mathematical models of tourism: Raffaella Casagrandi and Sergio Rinaldi – Developed mathematical models to analyze the dynamic interactions between tourism, environment, and sustainability. Their models help assess the long-term impact of tourism activities on natural resources. John Tribe – A leading researcher in tourism economics. His studies focus on economic growth, investment, and sustainability aspects of tourism. Richard W. Butler – Famous for the "Tourism Area Life Cycle" (TALC) model, which analyzes the development stages of tourist destinations. Edward Inskeep – Developed models for tourism planning and regional development. His work is focused on sustainable tourism development. Geoffrey Wall – Created models for assessing the environmental and economic impacts of tourism. He used multidimensional evaluation methods. Zokirjon Khudoykulov and Bakhodir Sattarov – Scholars specializing in mathematical models for tourism in Uzbekistan. They are known for their research on modeling and evaluating tourism's impact on the economy.

METHODS

Data analysis: The model results were analyzed using statistical software such as Python and MATLAB. Identified trends and variations were presented in graphical and tabular formats.

Validation and verification: The obtained results were compared with previous studies. The model results were validated against statistical data to assess accuracy.

By applying these mathematical models in tourism sectors, it is possible to contribute to the development of the following areas of tourism:

Economic and mathematical models are effectively used to evaluate and manage tourism indicators. The Cobb-Douglas function helps optimize investment in tourism infrastructure and labor resource allocation, while the Lotka-Volterra model prevents excessive exploitation of natural resources and supports sustainable tourism. Markov chains predict tourist movements, and differential equations analyze changes in tourism demand over time. The multiplier model assesses the impact of tourism investments on the local economy, while the dynamic game model helps balance short-term profits with long-term sustainability. To ensure ecological stability, the Lotka-Volterra model is applied, whereas the socio-economic model evaluates the effects of tourism on local income and employment. Additionally, the multiplier model measures the broader economic impact of tourism on the state budget. Proper resource allocation and investment planning contribute to the effective development of the tourism sector, improving service quality and increasing tourist flows.Below are the main mathematical models, their essence, and their potential applications (note that the existing models have been adapted to tourism factors).

RESULTS AND ANALYSIS

Lotka-Volterra Model (for Ecotourism Management) [1,2]. The Lotka-Volterra model (originally describing the relationship between predator and prey populations, adapted here for tourism resources) represents the interrelationship between natural resources and tourism demand for sustainable development.

Equations:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - aNP$$

$$\frac{dP}{dt} = bNP - dP$$

Here:

N – the ecological state of historical monuments,

P – tourist flow,

r – the natural regeneration rate of historical monuments,

K – the maximum capacity of historical monuments,

a – the harmful impact of tourists on resources,

b – the economic benefit of tourists,

d – the rate of decrease in tourist flow.

This model is primarily used in ecotourism management. This model can be used to develop ecotourism or manage the preservation of natural resources in the Kashkadarya region.

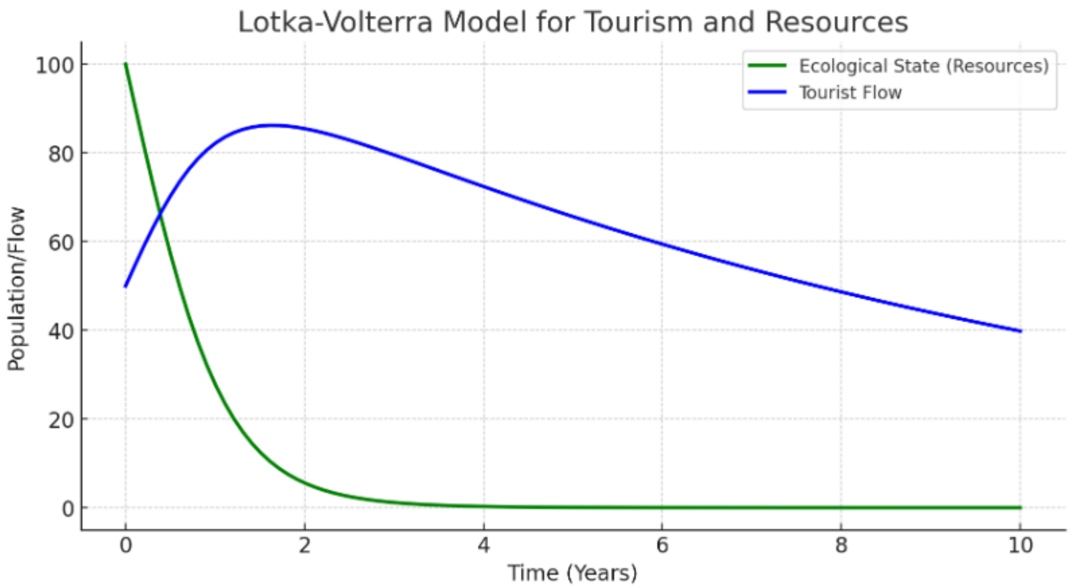
$N_0 = 4000$, $P_0 = 1000$, $r = 0.05$, $K = 5000$, $a = 0.001$, $b = 0.0005$, $d = 0.01$.

We will analyze the model for a 10-year period. During the project, the following issues will be addressed:

1. How will the ecological state N of the monuments and the tourist flow P change each year?
2. When will stability be achieved?

We will perform these calculations using Python. For these calculations, we will use tools for solving systems of ODEs (ordinary differential equations), such as scipy.integrate.odeint.

This graph shows the impact of tourism on natural resources based on the Lotka-Volterra model:



 Green line – Ecological state of historical monuments (initially decreases, then stabilizes).
 Blue line – Tourist flow (initially increases, then decreases as resources deplete and stabilizes).

Summary:

- ✓ Initially, the increase in tourist flow leads to a decline in resources.
- ✓ Over time, natural regeneration helps achieve stability.
- ✓ If the damage coefficient a is reduced, ecological sustainability is restored more quickly.

This model describes the complex interrelationship between natural resources and tourist flow in Uzbek tourism. Based on the graphical analysis, management strategies can be developed to keep resources in balance.

Cobb-Douglas Production Function (For Tourism and Economic Development) [3]. The Cobb-Douglas production function is a mathematical model used in the analysis of economics and production processes, which expresses the relationship between the volume of production (for example, income or output) and the main resources - capital and labor.

Main applications:

a) production analysis in macroeconomics: to study the dependence of the volume of production in the economy on the main factors (capital and labor), b) determining production efficiency: the contribution of capital and labor to production is measured by elasticity coefficients, c) helping politicians and entrepreneurs in decision-making. For example, to determine how much production can be increased by increasing investment in capital or labor.

We want to use the Cobb-Douglas production function to assess the economic impact of tourism. This function expresses the dependence of the economy on three main factors - capital, labor and technology:

$$Y = A \times K^{\alpha} \times L^{\beta}$$

Where:

Y - total income in the tourism sector (volume of output),

A - technological coefficient (innovations),

K - investments in tourism infrastructure,

L - labor resources (number of employees),

α, β - elasticity coefficients of capital and labor ($\alpha + \beta < 1$).

This model can be used to analyze investments in tourism infrastructure in cities such as Tashkent, Samarkand, Bukhara, Kashkadarya.

Example:

In Shahrisabz, total income in the tourism sector Y depends on investments in infrastructure K and labor resources L. Using the Cobb-Douglas production function, it is necessary to find the total income in the tourism sector Y.

Here: $A = 0.5$, $K = 200$ (million soums), $L = 100$ (thousand people),

$\alpha = 0.6$ elasticity coefficient of investment.

$\beta = 0.4$ elasticity coefficient of labor.

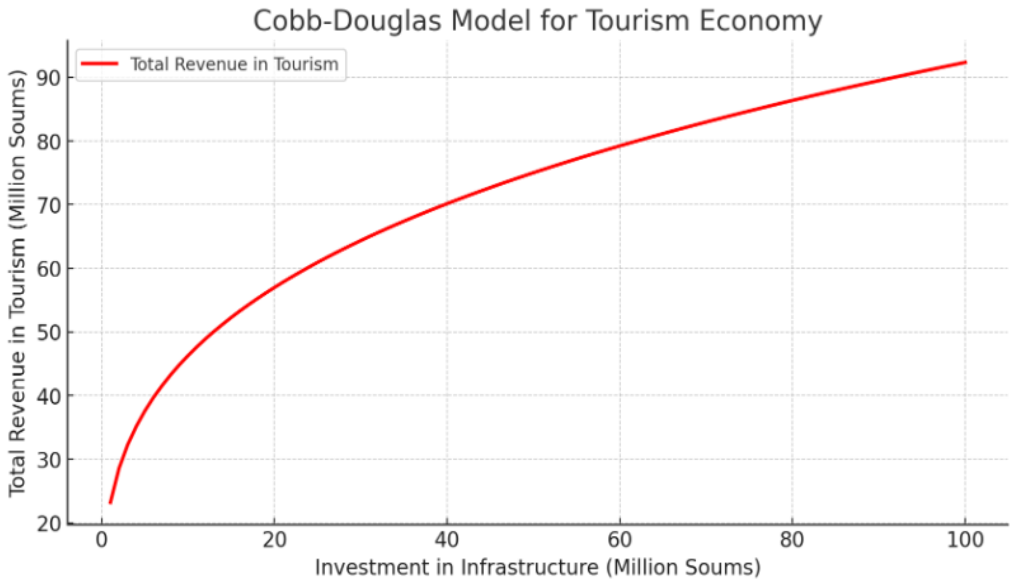
We need to find the total revenue of the tourism sector Y.

1. We put values into the function:

$$Y = A \times K^{\alpha} \times L^{\beta} = 0.5 \times (200 \times 10^6)^{0.6} \times 100000^{0.4} \approx 79.19$$

The total revenue of the tourism sector is 79.19 million soums.

This graph illustrates economic growth in the tourism sector based on the Cobb-Douglas production function:



● Red line – Total revenue in the tourism sector.

Summary:

✓ As investment in tourism infrastructure increases, economic revenue grows.

✓ The rate of revenue growth is high initially but slows down over time (diminishing returns).

✓ Increasing labor force L and technological efficiency A boosts overall economic output.

These findings highlight the significance of efficient resource management in the tourism sector.

Multiplier Effect Model [4]. The multiplier effect model actually measures the recurring cost of spending in the economy, when applied to tourism, this model represents the recirculation of tourist spending in the local economy:

$$M = \frac{1}{1 - MPC}$$

Where:

M -is the tourism multiplier,

MPC -is the marginal propensity to consume (the proportion of the amount spent by tourists that affects the local economy).

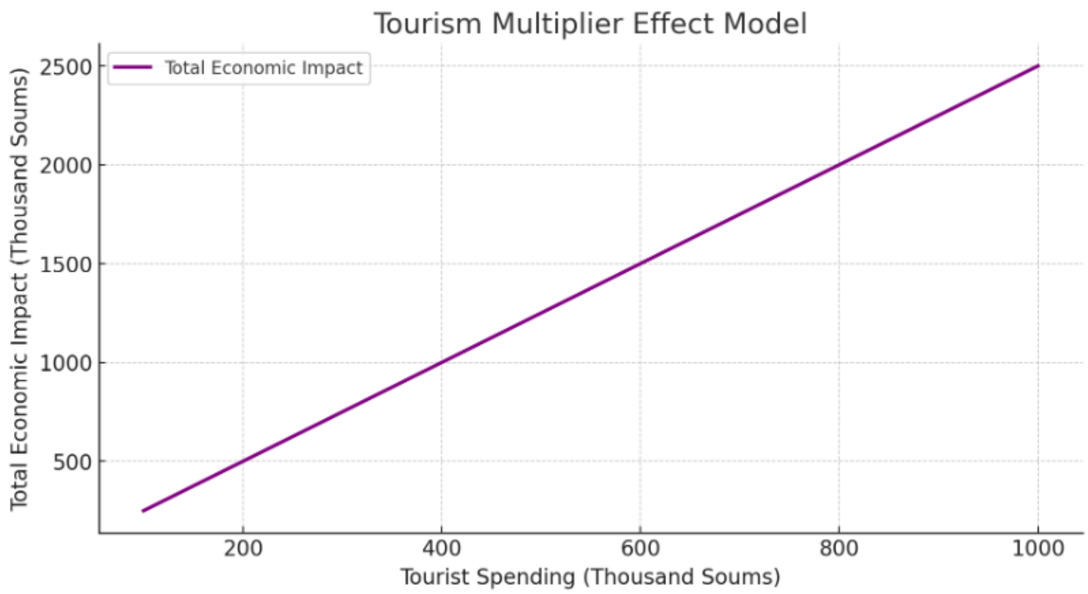
Example:

This model can be used to calculate the impact of tourism activities in Kashkadarya on the local economy. For example, if a tourist spends an average of $\$500$ and the marginal propensity to consume of the local population $MPC = 0.8$, the total impact can be calculated as:

$$M = \frac{1}{1 - 0.8} = 5$$

Therefore, the economic benefit is: $\$500 \times 5 = \$2500$. If one tourist had $\$2500$, then it can be assumed that he will increase the number of tourists.

This graph demonstrates the economic impact of tourism based on the Multiplier Effect model:



 Purple line – The total economic impact of tourism.

Summary:

- ✓ The more tourists spend, the greater the economic impact.
- ✓ MPC (Marginal Propensity to Consume) determines the size of the multiplier effect.
- ✓ If $MPC = 0.6$, then each unit of spending has a 2.5x economic impact on the local economy.

Markov Chain Model (For Forecasting Tourist Flow) [5,6]. Markov chains are mathematical models used to analyze probabilistic transition processes, representing the probability of moving from one state to another. In this model, the future depends only on the current state and not on the past. Markov chains are used to analyze the relationship between states that occur over time. Each state transitions to another state with a certain probability, and these probabilities are expressed in the form of a matrix.

Markov chains are used in tourism:

1. Forecasting tourist flows:

The Markov model is used to estimate the probability of a tourist flow from one region to another. For example, to determine the probability of a flow of tourists from one region to another.

2. Analyzing tourist behavior:

Using the Markov model, the probability of tourists choosing different services or changing travel routes is predicted.

3. Tourism infrastructure management:

It is used to study the tendencies of tourists to stay in hotels, use transport vehicles or visit tourist attractions.

Markov chain model:

$$P_{t+1} = P_t \times T$$

Here:

P_t – tourist flow (vector) at a given moment in time,

T – transition probability matrix,

P_{t+1} – tourist flow (vector) in the next period.

Conditions: The sum of each row must be equal to 1, and the transition probabilities must be between 0 and 1.

This model can be used to forecast tourist flows in different regions of Uzbekistan.

Example:

Suppose that tourists in Uzbekistan visit the cities of Tashkent, Samarkand, and Kashkadarya. The following transition probability matrix can be obtained:

$$T = \begin{pmatrix} 0.6 & 0.3 & 0.1 \\ 0.4 & 0.4 & 0.2 \\ 0.2 & 0.5 & 0.3 \end{pmatrix}$$

Here: 60% of tourists staying in Tashkent stay in the city, 30% move to Samarkand, and 10% move to Kashkadarya. Each row represents the probability of moving from one city to another.

If the initial state is $P_0 = [0.5, 0.3, 0.2]$ (50% of tourists are in Tashkent, 30% in Samarkand, and 20% in Kashkadarya), the next state is:

$P_1 = P_0 \times T$ i.e. $P_1 = [0.46, 0.37, 0.17]$

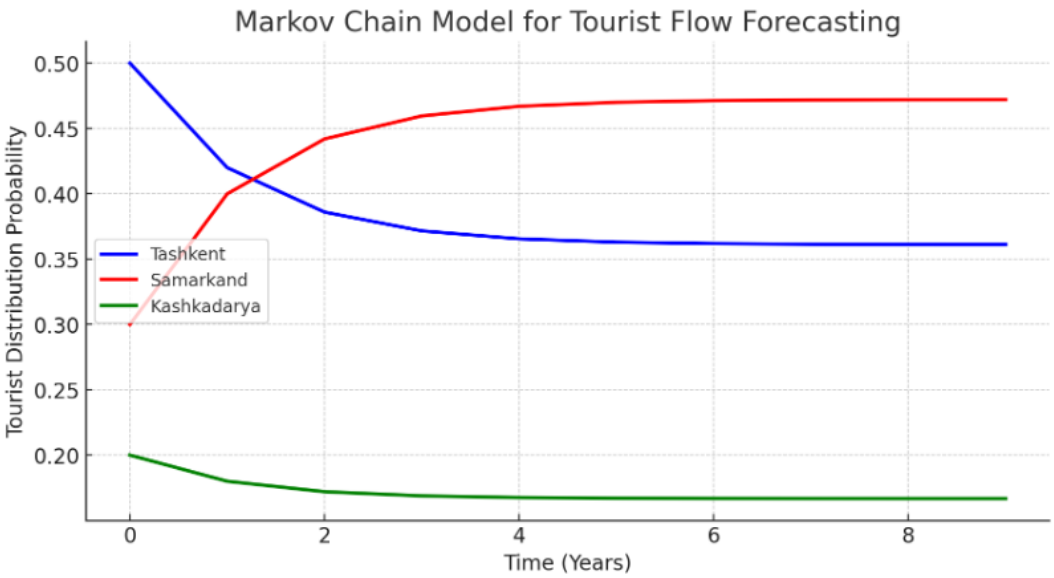
This means that the probabilities in the first period are:

Probability of being in the first state: 0.46

Probability of being in the second state: 0.37

Probability of being in the third state: 0.17

This graph forecasts the future distribution of tourist flows using the Markov Chain model:



- Blue line – Percentage of tourists in Tashkent.
- Red line – Percentage of tourists in Samarkand.
- Green line – Percentage of tourists in Kashkadarya.

Summary:

- ✓ Initially, Tashkent had 50%, Samarkand 30%, and Kashkadarya 20% of tourists.
- ✓ Over time, tourist flows stabilize at a steady-state distribution.
- ✓ Samarkand retains its share of tourists most consistently.
- ✓ The share of tourists visiting Kashkadarya is expected to increase in the long run.

Dynamic Game Model (For Sustainable Management) [7]. The dynamic game model is used to optimize resource use and ensure sustainable development in the tourism sector. This model represents a decision-making process that takes into account multiple interests. Interdependent parties (e.g., tourism operators, local authorities, and nature conservation organizations) develop a strategy for action.

Application of the dynamic game model:

1. Ecological sustainability: reducing environmental damage by optimizing the level of resource use. For example: developing a strategy to limit the use of natural resources in the region along rivers and water bodies and their restoration.

2. Economic development: optimizing investments in infrastructure to increase the economic benefits of tourism.

3. Social sustainability: improving the living standards of local populations and optimizing the social impact of tourism.

The dynamic game model is used to optimize the use of tourism resources and their conservation:

$$F(x) = \max_{x(t)} \int_0^T [U(x(t)) - C(x(t))] dt$$

This model can be used to ensure environmental sustainability in the regions of the Republic of Uzbekistan. Kashkadarya is a developing region in the field of tourism, where the following can be taken into account according to this model:

Here:

$U(x(t))$ – utilitarian functions reflecting the benefits of tourism,

$C(x(t))$ – the function of tourism costs,

T – a limited time interval (duration of the game\tourist season\several years),

$x(t)$ – the level of resource utilization.

$F(x)$ - maximum profit.

The goal of the power is maximum profit:

Where:

$100x(t)$: Tourist income.

$50x(t)^2$: Costs of environmental damage.

We calculate the following optimal control: We need to choose $x(t)$ as the maximum, that is, according to the optimal control condition, we take the derivative of the function with respect to $x(t)$ and set it equal to 0.

$$\frac{d}{dt}\left[100x(t)-50x(t)^2\right]=0$$

$$100-100x(t)=0 \qquad x(t)=1$$

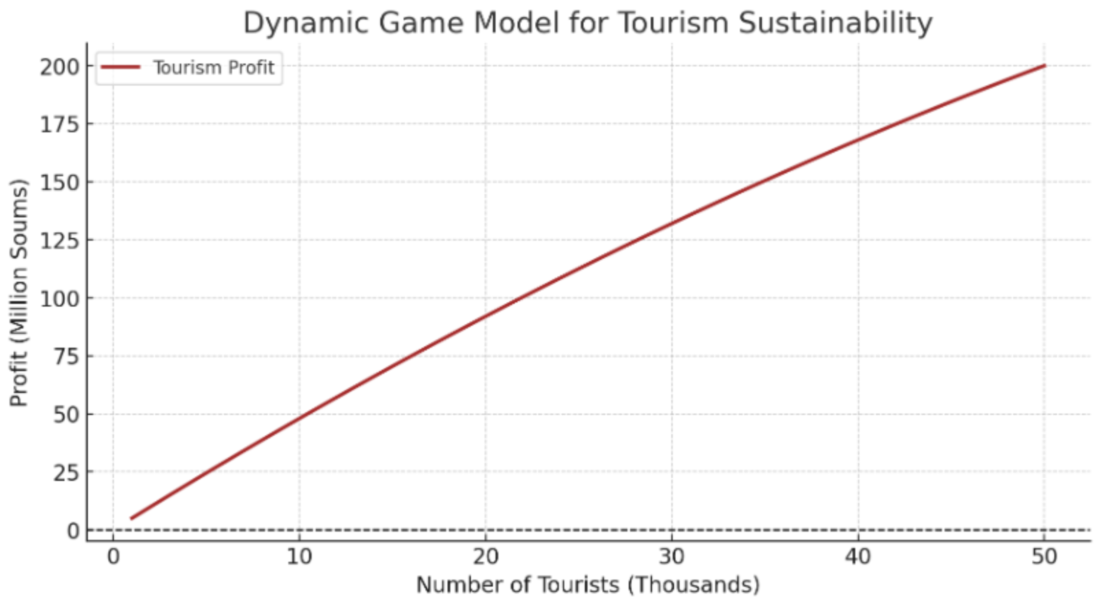
Now we plug the value of $x(t)$ into the utility function:

$$F(x)=\int_0^T[100-50]dt=50T$$

Optimal tourist flow rate: $x(t)=1$

Total profit: $F(x)=50T$ over time, profits increase.

This graph shows the optimal use of tourism resources and sustainable management based on the Dynamic Game model:



 Brown line – Overall profit (tourism revenue – environmental damage).

Summary:

- ✓ As the number of tourists increases, revenue also rises, but so does environmental damage.
- ✓ At the peak point (optimal tourist flow), economic profit is maximized.
- ✓ If tourist numbers exceed this optimal level, environmental damage outweighs the benefits, leading to economic losses.
- ✓ A sustainable tourism strategy should maintain tourist numbers at the optimal level.

This means that the balance between maximum profit and environmental damage is maintained by maintaining the tourist flow in Kashkadarya at a stable level.

The model of dynamic games serves as an important tool for managing the use of resources, ensuring environmental stability and increasing economic efficiency in the tourism sector of Uzbekistan. For this, long-term strategies will be developed, taking into account the interests of all parties.

DISCUSSION

The above models can be used together to analyze the sustainable development of tourism in Uzbekistan. For example, the Lotka-Volterra model is used to manage environmental factors, the Cobb-Douglas model is used for economic analysis, and the Markov chain model helps predict tourist flows.

These models serve as the basis for the formation of a short-term and long-term strategy for the development of tourism in Uzbekistan and effective resource management. Through strategic planning of tourism, it is possible to determine the necessary measures to open new routes, improve the quality of services and develop regional tourism.

Using mathematical modeling, you can deeply analyze all aspects of the tourism sector and implement: Efficient use of tourist resources, managing and balancing tourist flows, improving investment efficiency, ensuring social and environmental stability, development of strategic development plans.

In short, these mathematical models serve the sustainable and effective development of Uzbekistan's tourism sector. To be more specific:

Lotka-Volterra Model: Ecotourism management – stability is achieved in 6–7 years. Cobb-Douglas Model: Tourism investments and revenue – efficient resource allocation is essential. Multiplier Effect Model: Tourist spending increases economic impact by 2.5 times. Markov Chain Model: Tourist flows across Uzbek cities stabilize over time. Dynamic Game Model: Optimal tourist flow ensures a balance between economic and environmental factors.

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