American Journal Of Applied Science And Technology
(ISSN – 2771-2745)
VOLUME 04 ISSUE 02 Pages: 12-17
SJIF IMPACT FACTOR (2021: 5. 705) (2022: 5. 705) (2023: 7.063)
OCLC – 1121105677
Publisher: Oscar Publishing Services

Journal Website: https://theusajournals.com/index.php/ajast

Research Article

FULFILLMENT OF LOBACHEVSKY'S AXIOM IN EUCLIDEAN SPACE OF THE POINCARE INTERPRETATION OF LOBACHEVSKY'S GEOMETRY

Submission Date: February 05, 2024, Accepted Date: February 10, 2024,
Published Date: February 15, 2024
Crossref doi: https://doi.org/10.37547/ajast/Volume04Issue02-03

Fayzullaev Sherzod
Teacher At The Department Of Mathematics Teaching Methodology At Jizzakh State Pedagogical University, Uzbekistan

ABSTRACT

It is known that the Poincaré interpretation of Lobachevsky's geometry is used in solving many technical problems, in problems related to the theory of complex variable functions.

In this article, we show the Poincaré interpretation of the Lobachevsky plane, which is interpreted in a circle in a plane, using one circle of a two-section hyperboloid, using the method of spatial representation, and the Lobachevsky axiom and the results derived from it are also valid.

KEYWORDS

Lobachevsky's axiom, hyperbolic line, inversion, outer product, intersection, asymptotic cone, oscillating cone.

INTRODUCTION

Lobachevsky's geometry, the first non-Euclidean geometry, only after its interpretations appeared, did the scientific mind believe that these concepts are logically correct. There are many geometric interpretations of Lobachevsky's geometry. The most famous of them is the Kelly Klein interpretation, and one of the interpretations related to many technical issues and used in the theory of complex variables is the Poincaré interpretation.

The geometrical interpretations of Lobachevsky's geometry [3], [4] depend on how the basic concepts of planimetry, "point" and "straight line", are chosen from each other. Since the basic concepts are accepted without any definition, each teacher has his own idea about these concepts. In this article, we will get acquainted with the method of visualizing the elements of the Poincaré interpretation of Lobachevsky geometry using spatial forms.

Poincaré's interpretation of Lobachevsky's geometry, like his Kelly Klein interpretation, is represented by points inside a circle on a plane.

American Journal Of Applied Science And Technology
(ISSN – 2771-2745)
VOLUME 04 ISSUE 02 Pages: 12-17
SJIF IMPACT FACTOR (2021: 5. 705) (2022: 5. 705) (2023: 7.063)
OCLC – 1121105677
Publisher: Oscar Publishing Services

In our previous works, we considered the spatial representation of the Poincaré model of the Lobachevsky plane. In this case, we adopted the following initial concepts and adopted the following definition.

In the three-dimensional Euclidean space, let the surface π, which is one section of a two-section hyperboloid, and its asymptotic cone K be given. In this case, we choose the axis of symmetry of the cone K so that it coincides with its own axis, and the tip of the cone coincides with the origin of the coordinates. In that case, π surface and cone equations K are expressed in the following form.

$$\pi: \quad x^2 + y^2 - z^2 = 1$$
$$V: \quad x^2 + y^2 - z^2 = 0$$

In such a selection method, z=1 is the plane, π is the projection plane for the surface, and intersects with the cone K on a circle with the same radius $x^2 + y^2 = 1$ (Fig. 3).

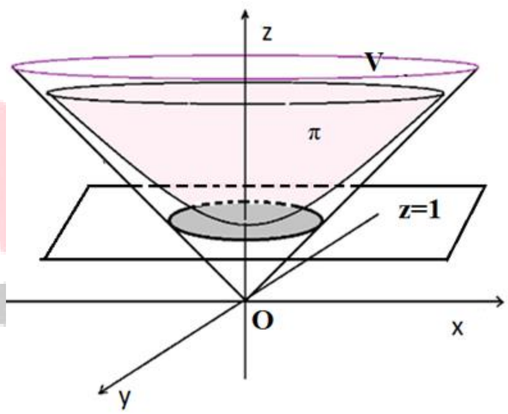

Figure 3

In this article, as in [2], we consider the points of the hyperboloid as the points of the Lobachevsky plane. In addition, let us denote by the set of conic cones $K(W(O_1))$whose ends are at the origin of the coordinates, and when all z=1 planes intersect, forming a circle, and these circles intersect the hyperboloid and are mutually orthogonal to the circle whose radius is equal to one. Let's look at the cone K belonging to this set $K(W(O_1))$. Naturally, this cone will be obtuse.

Theorem. Any conic generator $K \in K\left(W(O_1)\right)$ cuts one segment of the bisector hyperboloid π along the hyperbolic line l.

Proof. Since the generators of the cone K are orthogonal to the unit circle in the z=1 plane, it is natural that it intersects one segment $z > 0$ of the two-section hyperboloid π. We called this section hyperbolic. Because this section asymptotically approaches the V cone generators passing through the points C and D, which are the points of the circle section. This follows from the fact that the cone V is the asymptotic cone of the π hyperboloid.

Description. K'∈K(W(O_1)) conic generators, the line l formed by the intersection of the hyperboloid π with a circle is called a straight line in the interpretation of the Lobachevsky plane on the hyperboloid (Fig. 4).

American Journal Of Applied Science And Technology
(ISSN – 2771-2745)
VOLUME 04 ISSUE 02 Pages: 12-17
SJIF IMPACT FACTOR (2021: 5. 705) (2022: 5. 705) (2023: 7.063)
OCLC – 1121105677
Publisher: Oscar Publishing Services

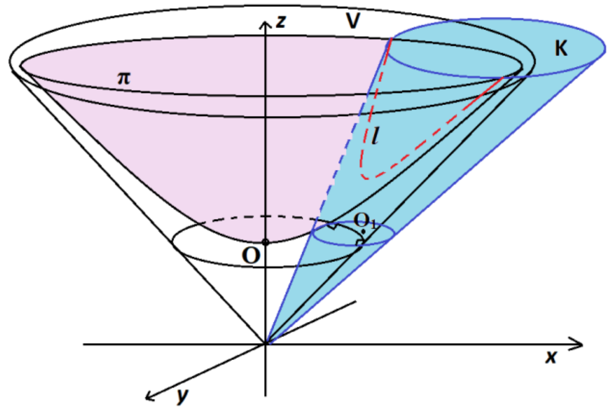

Figure 4

Now we show that the Lobachevsky axiom and the results derived from it are appropriate for the spatial representation of the Poincaré interpretation of the Lobachevsky plane.

Let us be given a regular cone $K' \in K(W(O_1))$and a point M that does not belong to this cone and lies on the hyperboloid. Let $K_1, K_2 \in K(W(O_1))$be given by such large cones. Let the point M belong to the generators of these obtuse cones. The obtuse cone $K' \in K(W(O_1))$ lies inside both obtuse cones $K_1, K_2 \in K(W(O_1))$and has no point in common with their generators (Fig. 8).

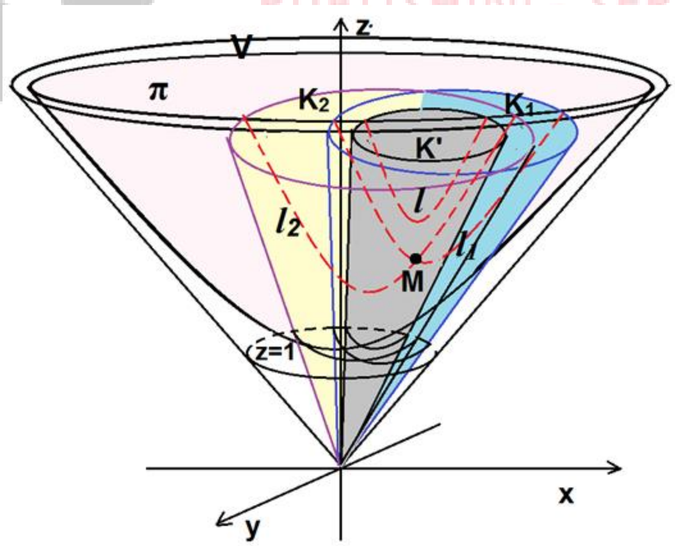

Figure 8

American Journal Of Applied Science And Technology
(ISSN – 2771-2745)
VOLUME 04 ISSUE 02 Pages: 12-17
SJIF IMPACT FACTOR (2021: 5.705) (2022: 5.705) (2023: 7.063)
OCLC – 1121105677
Crossref doi Google Scholar WorldCat MENDELEY
Publisher: Oscar Publishing Services

Here, the intersection of the hyperboloid π with the cone generators $K' \in K(W(O_1))$forms the Lobachevsky straight line. The Lobachevsky straight lines l_1 and l_2 formed by the intersection of the generators of the obtuse cones $K_1, K_2 \in K(W(O_1))$and π with the hyperboloid are formed and they pass through the point M. From this, it is possible to draw two straight lines l_1 and $l_2$that do not intersect with this straight line through a straight line l and a point M not lying on it.

So, the Lobachevsky axiom is fulfilled for the spatial representation of the Poincaré interpretation of the Lobachevsky plane.

Now we will consider the following conclusions from Lobachevsky's axiom.

1-Conclusion. An infinite number of straight lines that do not intersect with the straight line l can be drawn through the point M that does not lie on the given straight line l in the plane.

We show that this result is valid for the spatial representation of the Poincaré interpretation of the Lobachevsky plane.

We are given a regular cone $K' \in K(W(O_1))$ and a point M that does not belong to this cone and lies on the hyperboloid. Let there be hypercones $K_1, K_2 \in K(W(O_1))$ whose generators pass through the point M, both of which include the hypercone $K' \in K(W(O_1))$. Let's consider the obtuse cones $K_3, K_4 \in K(W(O_1))$such that the obtuse cone K_3contains the obtuse cone K_1, and the obtuse cone K_4contains the obtuse cone K_2, and at the point M have it. Then $K_3, K_4 \in K(W(O_1))$and the Lobachevsky straight lines l_3 and l_4 formed by intersection of the hyperboloid π pass through the point M and the straight line l does not lie passes through vertical angles. These straight lines l_3 and l_4 also do not intersect with the straight line l. Because in order to intersect with the straight line l, the straight line l_3 must cross the straight line l_1 at some point N. This leads to the violation of Euclid's 1st postulate. Likewise, the straight line l_4 does not intersect with the straight line l (Fig. 9).

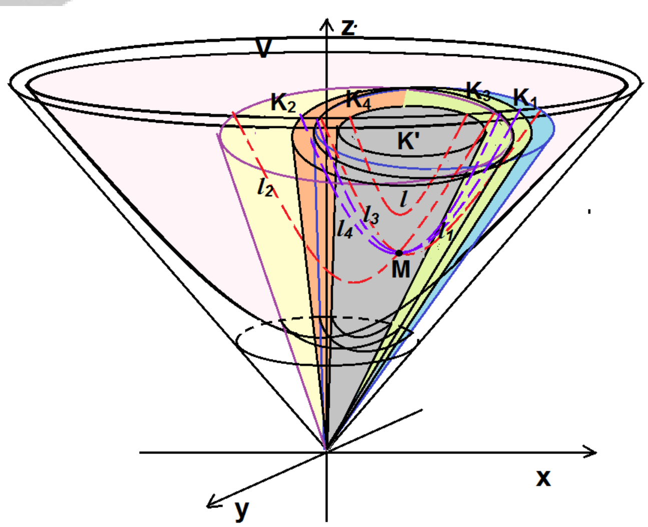

Figure 9

From this it can be concluded that infinitely many straight lines can be drawn through the point M, which is not on the straight line l, and does not intersect with the straight line l. Because you can get as many cones as you like.

2-Conclusion. Two parallel straight lines l can be drawn to a given straight line through a point that does not lie on it.

In order to show that this result is also valid, we are given a regular cone $K' \in K(W(O_1))$and a point M that does not belong to this cone and lies on the hyperboloid. Let $K_1, K_2 \in K(W(O_1))$be given cones, whose generators pass through the point M, both $K' \in K(W(O_1))$ and the generator of the asymptotic cone V. The Lobachevsky straight lines l_1 and l_2 formed by the intersection of these obtuse cones and π hyperboloid are parallel to the straight line l. The reason is that we showed above what parallel straight lines look like (Fig. 10).

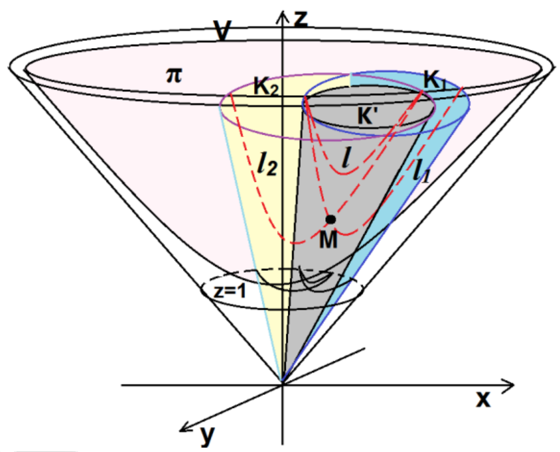

Figure 10

Therefore, the straight lines l_1 and l_2 are straight lines l passing through the point M and parallel to the straight line l.

CONCLUSION

In conclusion, it is possible to say that the Poincaré interpretation of the Lobachevsky plane in Euclidean space is the circle formed by the intersection of the two-section hyperboloid with one section and the asymptotic cone with the $z = 1$ plane, which forms the orthogonal circles formed by the intersection of the generators of this ellipse as long as the Lobachevsky axiom is fulfilled for straight lines and hyperboloid points formed by the intersection of convex cones.

REFERENCES

1. Sh. U. Fayzullaev. Puankare talqinining fazoviy tasviri. "Zamonaviy matematikaning nazariy asoslari va amaliy masalalar" Respublika ilmiy-amaliy anjumani materiallari to'plami. Andijon. 28-mart 2022 yil. I-qism.
2. I. M. Hatamov, SH. U. Fayzullaev. Lobachevskiy tekisligining gipervoloid ustidagi talqini. Fizika, matematika va informatika. Ilmiy – uslubiy jurnal. Toshkent – 2019 yil. 1-son.

American Journal Of Applied Science And Technology
(ISSN – 2771-2745)
VOLUME 04 ISSUE 02 Pages: 12-17
SJIF IMPACT FACTOR (2021: 5. 705) (2022: 5. 705) (2023: 7.063)
OCLC – 1121105677
Publisher: Oscar Publishing Services

3. Н.В.Ефимов. Высшая геометрия. Москва. Физматлит. 2004
4. Н.Г.Подаева , Д.А. Жук. Лекции по основам геометрии. Елец: 2008г.
5. 5 В. В. Прасолов Геометрия Лобачевского Независимый Московский Университет Математический колледж МЦНМО 2014.