



## METHODS OF SOLVING SOME INDETERMINATE INTEGRALS

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Submission Date: January 01, 2024, Accepted Date: January 05, 2024,

Published Date: January 08, 2024

Crossref doi: <https://doi.org/10.37547/ajast/Volume04Issue01-05>

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### ABSTRACT

This article shows examples of solving integrals known from the course of mathematical analysis, as well as methods of solving given integrals using functions called Logarithmic integral function. In addition, the integral equations are simultaneously solved by the method of integration by pieces into the differential. In turn, these types of solved examples are very important instructions for students of mathematics, physics and engineering.

### KEYWORDS

Logarithmic integral equation, indefinite integral, integration by pieces, differential, constant number, function, hyperbolic functions.

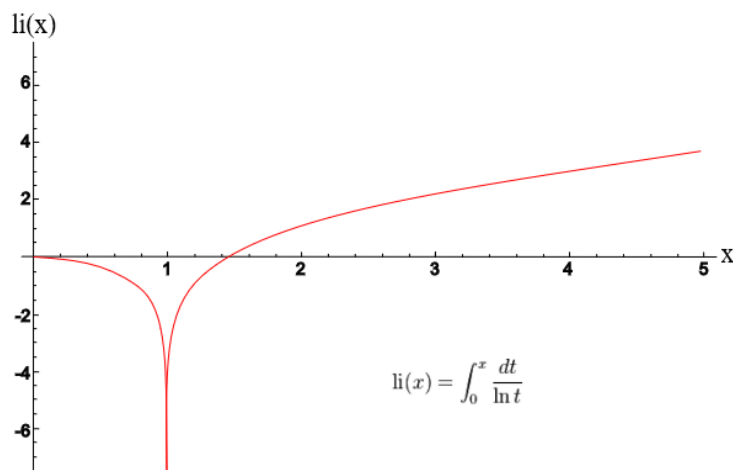
### INTRODUCTION

We know that the problem of calculating some indefinite integrals in the course of mathematical analysis becomes very complicated and incalculable, in these cases we have to introduce some definitions and solve the integrals. One such definition is the concept of logarithmic integral functions and hyperbolic functions. Let's first give information about the concept of logarithmic integral function. Usually, functions of this type are called logarithmic or integral logarithmic functions and are denoted by the term " $li(x)$ ". This function plays a key role in solving examples of physics and mathematics and number

theory, because this function is very important for approximate calculations.

$$li(x) = \int \frac{1}{\ln x} dx$$

We introduce the graph of the logarithmic integral function and some inequalities related to it, the Maple program was used to draw the graph, which was certainly very useful in drawing the graph of the function. We also get some results by comparing the function  $li(x)$ .



This function, in turn, can be compared as follows, i.e

$$li(x) = O\left(\frac{x}{\ln x}\right)$$

Here is an O (capital O symbol) and it looks like this

$$lix \sim \frac{x}{\ln x} \sum_{k=0}^{\infty} \frac{k!}{(\ln x)^k}$$

The above comparisons help us solve calculus examples, for example the Riemann Hypothesis.

$$\frac{lix}{\frac{x}{\ln x}} \sim 1 + \frac{1}{\ln x} + \frac{2}{(\ln x)^2} + \frac{6}{(\ln x)^3} + \dots$$

$$li(x) - \frac{x}{\ln x} = O\left(\frac{x}{(\ln x)^2}\right)$$

From this we arrive at the following inequality, i.e

$$1 + \frac{1}{\ln x} < \frac{li(x) \ln x}{x} < 1 + \frac{1}{\ln x} + \frac{3}{(\ln x)^2}$$

From this inequality,  $\ln x \geq 11$  for all  $x$ .

Since we used some hyperbolic functions in the calculation of the integral in the examples, we give the following information.

## Hyperbolic functions

Hyperbolic functions belong to the family of elementary functions, are determined by the exponential function and are closely related to trigonometric functions.

Below is information about these features:

Hyperbolic sine function–  $shx = \frac{e^x - e^{-x}}{2}$ ;

Hyperbolic cosine function –  $chx = \frac{e^x + e^{-x}}{2}$ ;

Hyperbolic tangent function–  $thx = \frac{shx}{chx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ ;

Hyperbolic cotangent function–  $cthx = \frac{chx}{shx} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ ;

A hyperbolic sequence function–  $schx = \frac{1}{chx} = \frac{2}{e^x + e^{-x}}$ ;

Hyperbolic cosecant function–  $cschx = \frac{1}{shx} = \frac{2}{e^x - e^{-x}}$ .

**Example 1.** Calculate the given integral for all (a>0) numbers

$$dx \int \frac{e^{ax}}{x}$$

We use the logarithmic integral function to calculate this integral.

$$li(x) = \int \frac{1}{\ln x} dx.$$

In that case, the given integral is solved using the differential subsumption method.

$$\begin{aligned} \int \frac{e^{ax}}{x} dx &= \int \frac{1}{ax} d(e^{ax}) = \int \frac{1}{\ln e^{ax}} d(e^{ax}) = [e^{ax} = t] = \int \frac{1}{\ln t} dt = \\ &= li(t) + C = li(e^{ax}) + C. \end{aligned}$$

The use of the li(x) function plays a key role in solving the above example and is very important in calculating the integral.

**Example 2.**

$$\int \left(1 - \frac{2}{x}\right)^2 \cdot e^x dx$$

When calculating this indefinite integral, we use the logarithmic integral function and the above equality  $\int \frac{e^x}{x} dx = li(e^x) + C$ .

$$\begin{aligned} \int \left(1 - \frac{2}{x}\right)^2 \cdot e^x dx &= \int \left(1 - \frac{4}{x} + \frac{4}{x^2}\right) e^x dx = \int e^x dx - 4 \int \frac{e^x}{x} dx + 4 \int \frac{e^x}{x^2} dx = \\ &= e^x - 4li(e^x) - 4 \int e^x d\left(\frac{1}{x}\right) + C = e^x - 4li(e^x) - 4\left(e^x \cdot \frac{1}{x} - \int \frac{e^x}{x} dx\right) + C = \\ &= e^x - 4li(e^x) - 4\left(e^x \cdot \frac{1}{x} - li(e^x)\right) + C = e^x - 4li(e^x) - 4e^x \cdot \frac{1}{x} + 4li(e^x) + C = e^x - 4e^x \cdot \frac{1}{x} + C \end{aligned}$$

Similarly, we calculate the following integral:

**Example 3.**

$$\int \left(1 - \frac{1}{x}\right) e^{-x} dx$$

When calculating this indefinite integral, we use the logarithmic integral function and the equality  $\int \frac{e^{-x}}{x} dx = li(e^{-x}) + C$  shown in the first example:

$$\int \left(1 - \frac{1}{x}\right) \cdot e^{-x} dx = \int \left(e^{-x} - \frac{e^{-x}}{x}\right) dx = \int e^{-x} dx - \int \frac{e^{-x}}{x} dx = e^{-x} - li(e^{-x}) + C$$

In addition, the logarithmic integral function is also important for solving the following examples, it can be seen from the examples solved above that

$$\int \frac{e^{ax}}{ax^2 + bx + c} dx \quad (b^2 - 4ac \geq 0), \int \frac{e^{ax}}{ax + b} (a \neq 0)$$

integrals of this form can be calculated using the logarithmic integral function.

**Example 4.**

Calculate the given integral:

$$\int \frac{e^{2x}}{x^2 - 3x + 2} dx$$

When calculating this indefinite integral, we also rely on the equality shown in Example 1:

$$\int \frac{e^{2x}}{x^2 - 3x + 2} dx = \int e^{2x} \left( \frac{1}{x-2} - \frac{1}{x-1} \right) dx = e^4 \int \frac{e^{2(x-2)}}{x-2} d(x-2) -$$

$$-e^2 \int \frac{e^{2(x-1)}}{x-1} d(x-1) = e^4 \cdot \text{li}e^{2(x-2)} - e^2 \cdot \text{li}e^{2(x-1)} + C$$

Similarly, we calculate the following integrals:

### Example 5.

Calculate the given integral:

$$\int \frac{xe^x}{(x+1)^2} dx$$

Using the equation in Example 1, we solve:

$$\int \frac{xe^x}{(x+1)^2} dx = \int \frac{(x+1)e^x}{(x+1)^2} dx - \int \frac{e^x}{(x+1)^2} dx =$$

$$= \int \frac{e^{x+1}}{x+1} d(x+1) + \int e^x d\left(\frac{1}{x+1}\right) = \text{li}(e^{x+1}) + e^x \cdot \frac{1}{x+1} - \frac{1}{e} \int \frac{e^{x+1}}{x+1} d(x+1) + C$$

$$= \text{li}(e^{x+1}) + \frac{e^x}{x+1} - \frac{1}{e} \cdot \text{li}(e^{x+1}) + C = \frac{e^x}{x+1} + \left(1 - \frac{1}{e}\right) \text{li}(e^{x+1}) + C$$

Thus, we use the logarithmic integral function to solve the following final integral.

$$\int e^{2x} \cdot \frac{x^4}{(x-2)^2} dx = \int e^{2x} \left( x^2 + 4x + 12 + \frac{32}{x-2} + \frac{16}{(x-2)^2} \right) dx =$$

$$= \int e^{2x} (x^2 + 4x + 12) dx + 32e^4 \int \frac{e^{2(x-2)}}{x-2} dx - 16 \int e^{2x} d\left(\frac{1}{x-2}\right) =$$

$$= \frac{e^{2x}}{2} \left( x^2 + 3x + \frac{21}{2} \right) + 32e^4 \text{li}e^{2(x-2)} - 16 \left( e^{2x} \cdot \frac{1}{x-2} - 2e^4 \int \frac{e^{2(x-2)}}{x-2} d(x-2) \right) + C =$$

$$= \frac{e^{2x}}{2} \left( x^2 + 3x + \frac{21}{2} \right) + 32e^4 \text{li}(e^{2(x-2)}) - 16e^{2x} \cdot \frac{1}{x-2} + 32e^4 \text{li}(e^{2(x-2)}) + C =$$

$$= \frac{e^{2x}}{2} \left( x^2 + 3x + \frac{21}{2} - 32 \right) + 64e^4 \text{li}(e^{2(x-2)}) + C$$

### CONCLUSION

So, in the calculation of integrals, we come across certain types of integrals, and in the calculation of

these types of integrals, we used notations that bring us convenience. In addition, we use the  $\text{li}(x)$  function to calculate complex integrals of the form

$\int \frac{e^{ax}}{ax^2+bx+c} dx$  ( $b^2 - 4ac \geq 0$ ). In addition, the logarithmic integral function makes it much easier for students to calculate this type of integrals.

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