

Statistical Analysis And Forecasting Of The Dynamics Of Harvested Grapes In The Republic Of Uzbekistan

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Abstract: Observations of phenomena whose characteristics change over time generate an ordered sequence known as a time series. In this research, a statistical time series test was applied to examine the statistical regularity of the average yearly volume (y_t) of harvested grapes in the Republic of Uzbekistan, based on data from the Central Statistical Office of Uzbekistan for the period 2012–2024. Point and interval predictions (with a 95% confidence level) were constructed for the mean annual volume of harvested grapes in the country. The explicit forms of trend models were identified, and forecasts were made for the predicted grape harvest volumes in the coming years. Using the Durbin–Watson statistical test, it was established that the average volume of harvested grapes in Uzbekistan exhibits autocorrelation dependence.

Keywords: Discrete, grape, time series, volume, trend, seasonality, component, linear, least squares, normal distribution, hypothesis, autocorrelation, asymmetry, kurtosis.

Introduction: In almost every field, there exist phenomena that are important to the research in terms of their development and changes over time. For example, one may seek to predict the future based on knowledge of the past, to manage ongoing processes, or to describe the characteristic features of a series using a limited amount of information.

When analyzing time series, researchers rely on methods developed in mathematical statistics. Now, statistics offers a wide range of approaches for the analysis of time series [1, 2, 3, 4, 5].

METHODS

This study shows the processing and analysis of the average dynamic volume of harvested grapes in the Republic of Uzbekistan during the observation time from 2012 to 2024, represented as a discrete stationary

time series $\{y_t, t \in T\}$.

Generally, a time series $\{y_t, t \in T\}$ consists of four main components: the trend, fluctuations around the trend, the seasonal effect, and the random component.

This paper utilizes time series processing and analysis methods, including trend detection, normality and randomness tests, autocorrelation testing, the moving average method, the finite difference method, the least squares method, the Durbin–Watson statistical test, and other methods.

Using these methods of statistical time series analysis, point and interval estimates were performed for the mean annual volume of harvested grapes in the Republic of Uzbekistan. Explicit trend models were identified, and forecasts were made for the expected average grape harvest volumes in the following years.

In addition, several statistical hypotheses were tested. The study draws on the research of T. Anderson [1], M. Kendall et al. [2], N. P. Tikhomirov [3], B. A. Sulaimanov et al. [4], and A. A. Faiziev [5], who have made significant contributions to the study and analysis of dynamic time series.

RESULTS AND DISCUSSION

It is assumed that the average volume of annual grape harvesting in the Republic of Uzbekistan over the observation period (2012-2024) forms a stationary discrete-time series. Using the above-mentioned methods of statistical analysis of time series, we will

construct point, and interval estimates for the average volume of grapes harvested in Uzbekistan. The explicit form of the trend was identified, and forecasts were made for the expected grape harvest volumes in the coming years. In addition, several statistical hypotheses were tested to assess the reliability and adequacy of the obtained model.

In Figure 1, based on the experimental data (Table 1, column 3), the dynamics of the average annual volume of harvested grapes \bar{y}_t – in the Republic of Uzbekistan are illustrated geometrically in the following forms: a) as a scatter plot, b) as a histogram, and c) as a pie chart.

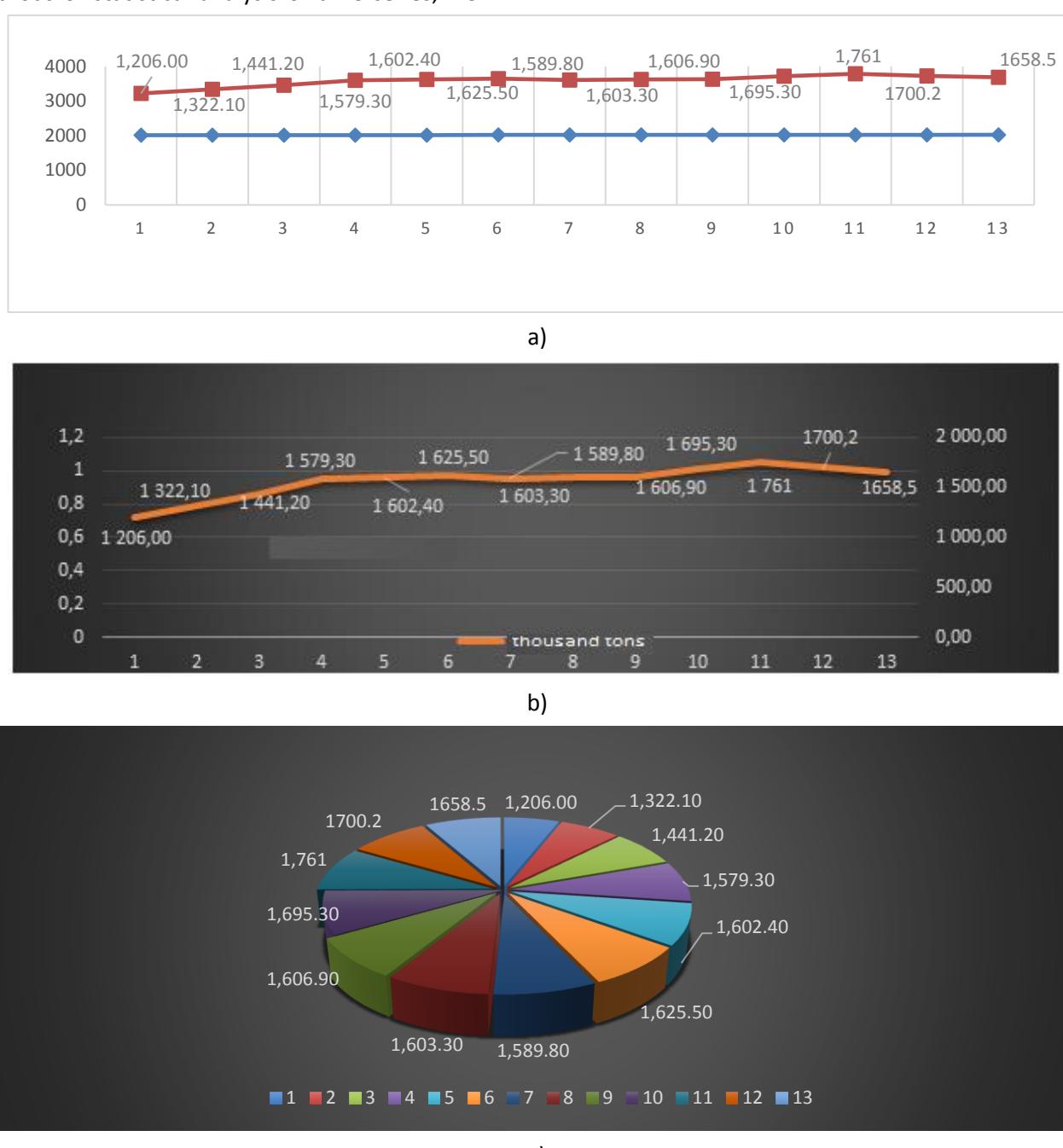


Figure 1

The geometric representation of experimental data on harvested grapes in the Republic of Uzbekistan in the

Cartesian coordinate system gives grounds, in the first approximation, to assume the hypothesis that the

trend part of the process, that is, the general direction of development of the random process, has a linear dependence

$$y(t) = a_1 t + a_0 \quad (1),$$

where the unknown parameters are determined by the least squares method, that is, based on experimental data, by solving the following system of normal equations:

$$\begin{cases} a_0 T + a_1 \sum t = \sum y_t \\ a_0 \sum t + a_1 \sum t^2 = \sum y_t t \end{cases} \quad (2)$$

Solving the system of equations (2) we determine

$$a_0 = \frac{1}{T} \sum y_t, \quad a_1 = \frac{1}{\sum t^2} \sum y_t t. \quad (3)$$

To estimate the unknown parameters a_0 and a_1 using experimental data, we will create calculation

table 2. Calculating data to determine the time series trend.

Table-1

1	2	3	4	5	6	7	8	9	10
N	Years of observations	y_t – thousand tons	Y_t^2	ΔY_t	ΔY_t^2	$\Delta^2 Y_t$	$\Delta^2 Y_t^2$	$\Delta^3 Y_t$	$\Delta^3 Y_t^2$
1	2012	1206,0	1454 436,0						
2	2013	1322,1	1747 948,4	116,1	13 479,2				
3	2014	1441,2	2 077 057,4	119,1	14 184,8	3,0	9,0		
4	2015	1579,3	2 494 188,5	138,1	19 071,6	19,0	361,0	16,0	256,0
5	2016	1602,4	2 567 685,8	23,1	533,6	-115,0	13 225,0	-134,0	17 956,0
6	2017	1625,5	2 642 250,3	23,1	533,6	0,0	0,0	115,0	13 225,0
7	2018	1589,8	2 527 464,0	-35,7	1 274,5	-58,8	3 457,4	-58,8	3 457,4
8	2019	1603,3	2 570 570,9	13,5	182,3	49,2	2 420,6	108,0	11 664,0
9	2020	1606,9	2 582 127,6	3,6	13,0	-9,9	98,0	-59,1	3 492,8
10	2021	1695,3	2 874 042,1	88,4	7 814,6	84,8	7 191,0	94,7	8 968,1
11	2022	1761	3 099 712,4	65,3	4 264,1	-23,1	533,6	-107,9	11 642,4
12	2023	1700,2	2 890 680,0	-60,4	3 648,2	-125,7	15 800,5	-102,6	10 526,8
13	2024	1658,5	2 750 622,3	-41,7	1 738,9	18,7	349,7	144,4	20 851,4
Total		20 391,1	32 278 785,6	452,5	66 738,3	-157,8	43 445,9	15,7	102 039,9

Harvested grapes in the Republic of Uzbekistan

$$\sum y_t = 20391,1 \text{ thousand tons}, \quad a_0 = \frac{1}{T} \sum y_t = \frac{20391,1}{13} = 1568,55$$

$$a_1 = \frac{1}{\sum t^2} \sum y_t t = \frac{6216}{182} = 34,15$$

Hence, the linear trend equation for the annual volume of harvested grapes in the Republic of Uzbekistan is

$$y(t) = 34,15 t + 1568,55 \quad (4)$$

Substituting the value $t = 1$ into equation (4), we find the expected forecast volume of grapes in the Republic of Uzbekistan in 2025, which will average **1,602,670** tons.

Using the statistical criteria cited ([1] – [5]), it was

determined that in equation (1) $y(t) = a_1 t + a_0$ the null hypothesis $H_0 : a_1 = 0$ is rejected and the alternative hypothesis $H_1 : a_1 \neq 0$ is accepted at the significance level $\alpha = 0.05$. Consequently, we may conclude that the mean annual volume of harvested

grapes in the Republic of Uzbekistan follows the linear trend (4). Next, using Table 2 we compute

$$V_k = \frac{\sum_{t=k}^T (\Delta^k y_t)^2}{(T-k) C_{2k}^k} \quad (5)$$

We determine the values of the coefficients of variation of the differences and establish that $V_1 \approx V_2 \approx V_3$. Therefore, first-order finite differences eliminate the linear trend.

For further research, we need to calculate the following finite differences based on the experimental data. We denote the finite differences:

$$\Delta Y_t = Y_{t+1} - Y_t, \Delta^2 Y_t = \Delta Y_{t+1} - \Delta Y_t, \Delta^3 Y_t = \Delta^2 Y_{t+1} - \Delta^2 Y_t$$

To calculate the final differences, we will create calculation table 2:

Table -2

1	2	3	4	5	6	7	8	9	10
N n/н	Years of observations	y_t – thousand tons	Y_t^2	ΔY_t	ΔY_t^2	$\Delta^2 Y_t$	$\Delta^2 Y_t^2$	$\Delta^3 Y_t$	$\Delta^3 Y_t^2$
1	2012	1206,0	1454 436,0						
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Total		20 391,1	32 278 785,6	452,5	66 738,3	-157,8	43 445,9	15,7	102 039,9

Next, we test the statistical hypothesis regarding the existence of autocorrelation in the annual grape

harvest volumes using the Durbin–Watson criterion:

$$d = \frac{\sum_{t=1}^{T-1} (Y_{t+1} - Y_t)^2}{\sum_{t=1}^T Y_t^2}. \quad (6)$$

Using Table 2 and equation (6), we calculate the following: $d_{\text{наб}}$

$$d_{\text{наб}} = \frac{66738,3}{32278785,6} = 0,0021.$$

$d_{\text{крит}} = 1,06$ establish that $d_{\text{наб}} = 0,0021 < d_{\text{крит}} = 1,06$.

Therefore, the Durbin-Watson criterion with 95% certainty proves that the average volume of grapes

Comparing them with the table value (see [5] – page 194, Appendix -9).

Consequently, the volume of grapes harvested in the Republic of Uzbekistan this year depends on the volume of previous and subsequent years.

Comparing them with the table value (see [5] – page 194, Appendix -9).

Testing the statistical hypothesis H_0 (7) about the

normality of \bar{y}_t – the average volume of grapes harvested annually in the Republic of Uzbekistan ([1] – [5]):

$$H_0 : P(\bar{y}_t < x) = \Phi_{a,\sigma}(x), \quad H_1: P(\bar{y}_t < x) \neq \Phi_{a,\sigma}(x) \quad (7)$$

the level of significance is accepted $\alpha = 0,05$ (cm. Table – 3).

Then, using the following formula (8) we construct

$$\bar{Y}_{T+i} - t(T-2; \alpha) \bar{\sigma}_y \leq a_0 + a_1(T+i) \leq \bar{Y}_{T+i} + t(T-2; \alpha) \bar{\sigma}_y \quad (8)$$

The value $t_{\text{крит.}} = t(T-2; \alpha)$ is determined using the student's table

$t_{\text{крит.}} = t(11; 0,05) = 2,45$ (см. [5] – стр. 190, application – 3).

Using formula (8) with a probability of 0.95, we construct an interval estimate for \bar{y}_T - the average volume of grapes harvested in the Republic of

interval estimates for \bar{y}_T – the average volume of grapes harvested in the Republic of Uzbekistan:

Uzbekistan

(1473,96; 1663,14) thousands of tons.

Based on sample data, using the x7.2019 software package and Excel [4,5], we calculate the numerical characteristics of \bar{y}_T – the average volume of grapes harvested in the Republic of Uzbekistan (Table 3).

Evaluation of the main parameters of the dynamic series:

Table 3

Sample characteristics	Statistical estimates for sample characteristics
The average volume of grapes harvested annually in the Republic of Uzbekistan (\bar{y}_T –thousands of tons)	1568,55
Dispersion	24533,68
Standard deviation σ_T	156,63
Coefficient of variation - v (%)	9,99 %
Asymmetry- A_S	-1,34
Excess- E_{K_S}	1,39
Sum	20391,1
Error of the mean \bar{y}_T , m_y	$m_y = \frac{\sigma_y}{\sqrt{n}} = 43,39$
Marginal error m'_y	$m'_y = t m_y = 2,18 \cdot 43,39 = 94,59$
Interval estimate (95%) $\bar{y}_T \pm t m_y$ for the volume of grapes	$\bar{y}_T \pm t m_y = 1568,55 \pm 94,59$ (1473,96; 1663,14) thousands of tons.
Testing a statistical hypothesis $H_0 : P(\bar{y}_t < x) = \Phi_{a,\sigma}(x)$ $H_1: P(\bar{y}_t < x) \neq \Phi_{a,\sigma}(x)$	Accepted with 95% guarantee of hypothesis H_0

Conclusions

Based on the above statistical analyses, the dynamics of the average annual harvested grape volume \bar{y}_t – in the Republic of Uzbekistan, considered as a discrete stationary time series with a confidence level of $\gamma = 0,95$ (Table 3), allow us to draw the following conclusions:

For the average annual harvested grape volume \bar{y}_t – in the Republic of Uzbekistan, a point estimate of **1,568.55** thousand tonnes and interval estimates of **(1,473.96; 1,663.14)** thousand tonnes were obtained.

1) The explicit form of the trend was determined, confirming its linear nature, expressed as:

$$y(t) = 34,17 t + 1568,55 ;$$

Using the Durbin–Watson criterion, it was established that the mean harvested grape volume in the Republic of Uzbekistan exhibits autocorrelation dependence, represented by:

$$Y_t = \rho Y_{t-1} + \varepsilon_t, \text{ где } \rho = \text{Cov}(Y_t, Y_{t+1}) = M[(Y_t - \bar{y}_t)(Y_{t+1} - \bar{y}_t)],$$

This indicates that the grape harvest volume in a given year depends on the volumes harvested in previous

and subsequent years.

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